Basic Topics in PROLOG

- Practical Matters
- A Brief Reminder
- Cases and Structural Induction
- Inputs and Outputs
- Context Arguments
  - Accumulator Passing
  - Last Call Optimization
  - Partial Data Structures
- Difference Lists
- Counters
- Backwards Correctness

Practical matters

Two Prologs are installed on the OSU Ling. Dept. UNIX machines:

- Sicstus:
  - starting:
    - at UNIX prompt: `prolog`
    - in Emacs: `M-x run-prolog`
  - manual (652 pages – so don't just print it!): links on course web page or `~dm/resources/manuals/sicstus/`
- SWI-Prolog:
  - starting: `pl`
  - loading graphical tracer: `?- guitracer.`
  - manual: links on course web page or `~dm/resources/manuals/swi-prolog/`

A brief reminder (1)

PROLOG (PROgrammation LOGique) invented by Alain Colmerauer and colleagues at Marseille in the early 70s. Parallel development in Edinburgh.

A PROLOG program is written in a subset of first order predicate logic:

- **constants** naming entities
  - Syntax: starting with lower-case letter, a number, or in single quotes
  - Examples: `twelve, a, q_1`
- **variables** over entities
  - Syntax: starting with upper-case letter or underscore
  - Examples: `A, This, _twelve, _`
- **predicate symbols** naming relations among entities
  - Syntax: predicate name starting with a lower-case letter with parentheses around comma-separated arguments
  - Examples: `father(tom,mary)`, `age(X,15)`

A brief reminder (2)

A PROLOG program consists of a set of *Horn* clauses:

- **unit clauses** (facts)
  - Syntax: predicate followed by a dot
  - Example: `father(tom,mary).`
- **non-unit clauses** (rules)
  - Syntax: rel_0 `:-` rel_1`, ...,` rel_n`.`
  - Example:

```
grandfather(Old,Young) :-
    father(Old,Middle),
    father(Middle,Young).
```

Basic use of arguments: Discriminate between cases

```
direction_adjective(north, boreal).
direction_adjective(south, austral).
direction_adjective(east, oriental).
direction_adjective(west, occidental).


abs_diff(X, Y, Diff) :-
     compare(R, X, Y),
     abs_diff(R, X, Y, Diff).
abs_diff(<,X,Y,Diff) :- Diff is Y-X.
abs_diff(>,X,Y,Diff) :- Diff is X-Y.
abs_diff(=,_,_,0).
```

Compound terms as data structure for recursive relations

To define (interesting) recursive relations, one needs a richer data structure than the constants used so far: *compound terms*.

- A compound term comprises a functor and a sequence of one or more terms, the argument. Atoms can be thought of as functors with arity 0.
- Compound terms are standardly written in prefix notation.

  Example: `bin_tree(s, np, bin_tree(vp,v,n))`

  Infix and postfix operators can also be defined, but need to be declared using op/3.

Lists as special compound terms

Lists are represented as compound terms.

- empty list: represented by the atom "[]"
- non-empty lists: symbol "." as binary functor `.(`*first*`,`*rest*`)`
  Example: `.(a, .(b, .(c, .(d,[]))))`

Special notations:

- `[` *element1* `|` *restlist* `]`
  Example: `[a | [b | [c | [d | []]]]]`
- `[` *element1* `,` *element2*`]` = `[` *element1* `| [`*element2* `| []]]`
  Example: `[a, b, c, d]`

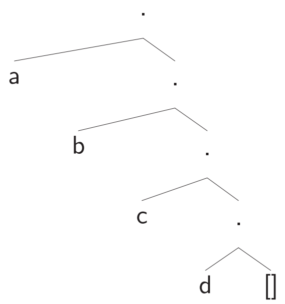
Four equivalent representations:

1. `.(a, .(b, .(c, .(d,[]))))`

2. `[a | [b | [c | [d | []]]]]`

3. `[a,b,c,d]`

4.



Structural induction

```
is_list([]).           % a) base/non-recursive case
is_list([_|Tail]) :-   % b) step/recursive/inductive case
     is_list(Tail).

% arithmetic_value(Expr,Value)
% is true when Expr represents an arithmetic expression and
% Value is its numeric value

arithmetic_value(c(N), N).            % a) base case
arithmetic_value(E+F, Value) :-       % b1) recursive case
     arithmetic_value(E,Eval),
     arithmetic_value(F,Fval),
     Value is Eval + Fval.
```

```
arithmetic_value(-F, Value) :-        % b2) recursive case
     arithmetic_value(F,Fval),
     Value is -Fval.
arithmetic_value(E-F, Value) :-       % b3) recursive case
     arithmetic_value(E,Eval),
     arithmetic_value(F,Fval),
     Value is Eval - Fval.
arithmetic_value(E*F, Value) :-       % b4) recursive case
     arithmetic_value(E,Eval),
     arithmetic_value(F,Fval),
     Value is Eval * Fval.
arithmetic_value(E/F, Value) :-       % b5) recursive case
     arithmetic_value(E,Eval),
     arithmetic_value(F,Fval),
     Value is Eval / Fval.
```

Why is this called *structural induction*?

- *induction*: defined recursively
- *structural*: recursion controlled by structure, not contents

Two things to watch out for:

- missing cases
- duplicate cases

  An example for intentional duplicate cases:

  ```
  member(X,[X|_]).
  member(X,[_|L]) :-
       member(X,L).
  ```

A closer look at arguments: Inputs and Outputs

In principle, any argument (or part of it) can be input or output:

```
birthday(byron,   date(feb,4)).
birthday(noelene, date(dec,25)).
birthday(richard, date(oct,11)).
birthday(clare,   date(sep,15)).


?- birthday(byron,Date).

?- birthday(Person, date(feb,4)).

?- birthday(Person, date(feb,Day)).
```

Predicates solving for particular arguments only

Built-in predicates involving arithmetic expressions

- *Expression* must be ground in evaluation of *Answer* `is` *Expression* (expression has one value, but same value for infinitely many expressions)
- Both arguments must be ground in comparisons: `E<F`, `E>F`, `E>=F`, . . .

  Predicates using these built-ins have specific inputs and outputs:

```
factorial(0,1).
factorial(N,N_Factorial) :-
     N > 0,
     M is N-1,
     factorial(M, M_Factorial),
     N_Factorial is M_Factorial*N.
```

Recursive predicates often require particular arguments to terminate.

Multiple output arguments

no output argument (true/false)

```
greater_than(X,Y) :- X < Y.
```

one output argument: min

```
min(X, Y, X) :- X < Y.
min(X, Y, Y) :- X >= Y.
```

two output arguments: min, max

```
min_and_max(X, Y, X, Y) :- X < Y.
min_and_max(X, Y, Y, X) :- X >= Y.
```

Order of arguments

Why a uniform ordering?

- clarity: consistency makes programs easier to understand
- efficiency: first argument indexing

Suggested ordering

- General rule: strict inputs < inputs-or-outputs < strict outputs
- Among strict inputs: templates < meta-arguments < streams < selectors/indices < collections < other strict inputs

---

Templates and meta-arguments

Template:

- Pattern for making/selecting things.
- Example: first argument of `findall/3`

```
?- findall(Month-Day, birthday(_Name,date(Month,Day)), Bag).

Bag = [feb-4,dec-25,oct-11,sep-15]
```

Meta-Argument:

- Term which stands for a goal.
- Example: argument of `call/1` or second argument of `findall/3`

Streams

- Terms representing open files
- Example: third argument of `open/3`

```
file_write :-
    open(myfile,write,MyStream), % modes: read/write/append
    write(MyStream,’output to file’),
    write(’output to screen (standard output)’),
    close(MyStream).

% simple case not using explicit streams
simple_file_write :-
    tell(myfile),
    write(’output to file’),
    told.
```

Selectors/Indices and Collections

Selectors/Indices:

- Terms which function like array subscripts.
- Example: first argument of `arg/3`

```
?- arg(3,p(a(n,o),b,c(m),d),X).
X = c(m)

?- functor(p(a(n,o),b,c(m),d),Functor,Arity).
Arity = 4, Functor = p
```

Collections:

- essentially every compound term can be used as a collection
- Example: second argument of `arg/3`

Other ordering guidelines

- sequence order: keep abstract sequences together (difference lists, accumulator pairs. . .)
- code/data consistency: e.g., `Head` < `Tail` since `[Head|Tail]`
- function direction: most general input first
  Example: `Term =.. List`
  (every `Term` corresponds to a `List`, but not vice versa)

```
?- p(a(n,o),b,c(m),d) =.. List.
X = [p,a(n,o),b,c(m),d]

?- Term =.. [1,a(n,o),b,c(m),d].
{TYPE ERROR: _169=..[1,a(n,o),b,c(m),d] -
 arg 2: expected atom, found 1}
```

The scope of variables

- There are no non-local variables in Prolog.
- Non-local variables are encoded as extra arguments of a predicate which are passed unchanged into the recursion.

```
% scale(SmallList,Multiplier,BigList)
% True if each element of SmallList multiplied by Multiplier
% is equal to the corresponding element of BigList.

scale([], _, []).
scale([X|Xs], Multiplier, [Y|Ys]) :-
     Y is X*Multiplier,
     scale(Xs, Multiplier, Ys).
```

```
% big_elements(FullList,SubList)
% True if SubList is the list of those elements of
% FullList which are bigger than 10, preserving order.

big_elements(Input,Output) :-
     big_elements(Input, 10, Output).

big_elements([], _, []).
big_elements([Nbr|Nbrs], Bound, Bigs) :-
     Nbr < Bound,
     big_elements(Nbrs, Bound, Bigs).
big_elements([Nbr|Nbrs], Bound, [Nbr|Bigs]) :-
     Nbr >= Bound,
     big_elements(Nbrs, Bound, Bigs).
```

Packaging contexts

```
context(conx(A,B,C,D),A,B,C,D).

context_a(conx(A,_,_,_),A).
context_b(conx(_,B,_,_),B).
context_c(conx(_,_,C,_),C).
context_d(conx(_,_,_,D),D).

c(...) :-
     init(...,A,B,C,D,...),
     context(Context,A,B,C,D),
     p(...,Context,...),
     ...
```

```
p(...,Context,...) :-
     ...
     context_a(Context,A),
     use_a(A),
     ...
     p(...,Context,...).

p(...,Context,...) :-
     ...
     context_b(Context,B),
     use_b(B),
     ...
     p(...,Context,...).
```

Accumulator passing

- There is no changing of variable values in Prolog.
- Two variables are used to store old and new value (*accumulator passing*).

```
len(List,Length) :-
     len(List, 0, Length).

len([], N, N).
len([_|L], N0, N) :-
     N1 is N0+1,
     len(L, N1, N).
```

```
rev(List, Reverse) :-
     rev(List, [], Reverse).

rev([], Reverse, Reverse).
rev([Head|Tail], Reverse0, Reverse) :-
     rev(Tail, [Head|Reverse0], Reverse).
```

Multiple accumulator pairs

One changed in each recursion

```
sum_pos_neg(List, Pos, Neg) :-
     sum_pos_neg(List, 0, Pos, 0, Neg).

sum_pos_neg([], Pos, Pos, Neg, Neg).
sum_pos_neg([X|Xs], Pos0, Pos, Neg0, Neg) :-
     X >= 0,
     Pos1 is Pos0+X,
     sum_pos_neg(Xs, Pos1, Pos, Neg0, Neg).
sum_pos_neg([X|Xs], Pos0, Pos, Neg0, Neg) :-
     X < 0,
     Neg1 is Neg0+X,
     sum_pos_neg(Xs, Pos0, Pos, Neg1, Neg).
```

Multiple accumulator pairs

Multiple changed in each recursion

```
sum_and_ssq(List, Sum, SSQ) :-
     sum_and_ssq(List, 0, Sum, 0, SSQ).

sum_and_ssq([], Sum, Sum, SSQ, SSQ).
sum_and_ssq([X|Xs], Sum0, Sum, SSQ0, SSQ) :-
     Sum1 is Sum0 + X,
     SSQ1 is SSQ0+X,
     sum_and_ssq(Xs, Sum1, Sum, SSQ1, SSQ).
```

Last call optimization/Tail-recursion optimization

- **Issue:** Before execution can enter a recursive call, it has to save the state of all variables.
- **Idea:** A recursive call as last goal in the body of a deterministic predicate can be turned into a jump.
- **Advantage:** A jump does not require saving the state of the variables before entering the recursion.

An example for ordinary recursion

```
slow_len([],0).
slow_len([_|Tail],N) :-
     slow_len(Tail,M),
     N is M+1.
```

How does the query `slow_len([a,b,c], X)` work?

1 Call: `slow_len([a,b,c], X)`

- Prolog tries to match it against `slow_len([],0)`, which fails.
- Prolog tries to match it against slow_len([_|$\mathtt{Tail}_1$], $\mathtt{N}_1$), which succeeds, binding $\mathtt{Tail}_1$=[b,c], $\mathtt{N}_1$=X.
- A stack frame is created, holding $\mathtt{N}_1$ and $\mathtt{M}_1$.
- Prolog now has the goal slow_len([b,c], $\mathtt{M}_1$).

---

2 Call: slow_len([b,c], $\mathtt{M}_1$)

- Prolog tries to match it against `slow_len([], 0)`, which fails.
- Prolog tries to match it against slow_len([_|$\mathtt{Tail}_2$], $\mathtt{N}_2$), which succeeds, binding $\mathtt{Tail}_2$=[c], $\mathtt{N}_2$=$\mathtt{M}_1$.
- A stack frame is created, holding N_2 and M_2.
- Prolog now has the goal slow_len([c], $\mathtt{M}_2$).

3 Call: slow_len([c], $\mathtt{M}_2$)

- Prolog tries to match it against `slow_len([], 0)`, which fails.
- Prolog tries to match it against slow_len([_|$\mathtt{Tail}_3$], $\mathtt{N}_3$), which succeeds, binding $\mathtt{Tail}_3$=[], $\mathtt{N}_3$=$\mathtt{M}_2$.
- A stack frame is created, holding $\mathtt{N}_3$ and $\mathtt{M}_3$
- Prolog now has the goal slow_len([],$\mathtt{M}_3$) .

---

4 Call: slow_len([], $\mathtt{M}_3$)

- Prolog tries to match it against `slow_len([], 0)`, which succeeds, binding $\mathtt{M}_3$=0.

3 Exit:

- Prolog returns to the third frame, and executes the goal $\mathtt{N}_3$ is $\mathtt{M}_3$+1, which succeeds, binding $\mathtt{N}_3$=1.
- The third stack frame is now released.

2 Exit:

- Prolog returns to the second frame, and executes the goal $\mathtt{N}_2$ is $\mathtt{M}_2$+1, which succeeds, binding $\mathtt{N}_2$=2.
- The second stack frame is now released.

---

1 Exit:

- Prolog returns to the first frame, and executes the goal $\mathtt{N}_1$ is $\mathtt{M}_1$+1, which succeeds, binding $\mathtt{N}_1$=3, which binds X=3.
- The first stack frame is now released.

A tail-recursion example using the optimization

```
len(List,Length) :-
     len(List, 0, Length).

len([], N, N).
len([_|L], N0, N) :-
     N1 is N0+1,
     len(L, N1, N).
```

How does the query `len([a,b,c], X)` work?

0 Call: `len([a,b,c], X)`
Prolog tries to match it against `len(List, Length)`, which succeeds, binding `List=[a,b,c]`, `Length=X`.

1a Jump: `len([a,b,c], 0, X)`
The clause `len([_|L, N0, N)` is selected, binding `L=[b,c]`, `N0=0`, `N=X`.

Ib Jump: `N1 is N0+1`
The goal `N1 is N0+1` is executed, binding `N1=1`.

2a Jump: `len([b,c], 1, X)`
The clause `len([_|L], N0, N)` is selected, binding `L=[c]`, `N0=1`, `N=X`.

2b Jump: `N1 is N0+1`
Execution of the builtin goal binds `N1=2`.

3a Jump: `len([c], 2, X)`
The clause `len([_|L], N0, N)` is selected, which binds `L=[]`, `N0=2`, `N=X`.

3b Jump: `N1 is N0+1`
Execution of the builtin goal binds `N1=3`.

4 Jump: `len([], 3, X)`
The clause `len([], N, N)` is selected, which binds `X=3`.

Partial Data Structures

An instance i of a recursively defined data type t is referred to as

- *proper* if i is not a variable and each of its argument of type t is proper
- *partial* or *incomplete* otherwise.

Examples:

- proper lists: `[]`, `[_,_,_]`
- partial lists: `X`, `[a|_]`, `[a|Rest]`

Classifying lists (an example for an accumulator pair)

```
is_proper_list(Term) :-
     classify_list(Term, proper, proper).

is_partial_list(Term) :-
     classify_list(Term, proper, partial).

is_a_list(Term) :-
     classify_list(Term, partial, partial).

classify_list(V, _, X) :- var(V), !, X=partial.
classify_list([],X,X).
classify_list([_|T],X0,X) :-
     classify_list(T, X0, X).
```

Why use partial data structures?

Partial data structures allow building results top-down:

```
append([], L, L).
append([H|T], L, [H|R]) :-
     append(T,L,R).
```

a bottom-up version (requires first argument is input):

```
append([], L, L).
append([H|T], L, X) :-
     append(T,L,R),
     X=[H|R].
```

```
?- append([1,2],[3,4],X).     % using "top-down" definition
        1      1 Call: append([1,2],[3,4],_274) ?

        2      2 Call: append([2],[3,4],_773) ? g
Ancestors:
        1      1 Call: append([1,2],[3,4],[1|_773])
        2      2 Call: append([2],[3,4],_773) ?

        3      3 Call: append([],[3,4],_1938) ? g
Ancestors:
        1      1 Call: append([1,2],[3,4],[1,2|_1938])
        2      2 Call: append([2],[3,4],[2|_1938])
        3      3 Call: append([],[3,4],_1938) ?

        3      3 Exit: append([],[3,4],[3,4]) ?
        2      2 Exit: append([2],[3,4],[2,3,4]) ?
        1      1 Exit: append([1,2],[3,4],[1,2,3,4]) ?
```

Walking through a tree vs. Unifying-in a pattern

```
path_data([], Tree, Datum) :-
     b_access(d, Tree, Datum).
path_data([Arc|Arcs], Tree, Datum) :-
     b_access(Arc, Tree, Dtr),
     path_data(Arcs, Dtr, Datum).

b_access(1, b(Lson,_,_),  Lson).
b_access(2, b(_,Rson,_),  Rson).
b_access(d, b(_,_,Datum), Datum).


dynamic_pattern(Path, Tree, Datum) :-
     path_data(Path, Pattern, Datum),
     Tree = Pattern.
```

Difference lists

- **Idea:** Carry around a partial data structure plus a reference to the holes in it.
- **Advantage:** The partial data structure can be extended by filling a hole with a (partial) data structure.

Example:

```
s(Phon0, Phon2) :-
     np(Phon0, Phon1),
     vp(Phon1, Phon2).

np([john|Hole],Hole).
np([laughs|Hole],Hole).
```

Counters: bottom-up

```
bup_ground(Term) :-
     nonvar(Term),
     functor(Term, _, Arity),
     bup_ground(Arity, Term).

bup_ground(0,_) :- !.    % no more elements to process
bup_ground(N, Term) :-
     arg(N, Term, Arg), % identify argument N
     bup_ground(Arg),   % check argument N
     M is N-1,
     bup_ground(M, Term).
```

Counters: bottom-up (if-then-else version)

```
bup_ground2(Term) :-
     nonvar(Term),
     functor(Term, _, Arity),
     bup_ground2(Arity, Term).

bup_ground2(N, Term) :-
     (N = 0 ->  % no more elements to process
         true
     ;
         arg(N, Term, Arg),     % process N
         bup_ground2(Arg),
         M is N-1,
         bup_ground2(M, Term)).
```

Counters: top-down

```
td_ground(Term) :-
     nonvar(Term),
     functor(Term, _, Arity),
     td_ground(0, Arity, Term).

td_ground(N,N,_) :- !.    % no more elements to process
td_ground(I, N, Term) :-
     J is I+1,
     arg(J, Term, Arg),  % identify argument J
     td_ground(Arg),        % check argument J
     td_ground(J,N,Term).
```

Counters: top-down (if-then-else version)

```
td_ground2(Term) :-
     nonvar(Term),
     functor(Term, _, Arity),
     td_ground2(0, Arity, Term).

td_ground2(I,N,Term) :-
     (I < N ->
         J is I+1,
         arg(J, Term, Arg),
         td_ground2(Arg),       % process element J
         td_ground2(J,N,Term)
     ;   true % I = N, no more items to process
     ).
```

Counters: bisection

```
bi_ground(Term) :-
     nonvar(Term),
     functor(Term, _, Arity),
     bi_ground(1, Arity, Term).

bi_ground(L, U, Term) :-
     L<U, !,
     M is (L+U)//2,
     N is M+1,
     bi_ground(L, M, Term),
     bi_ground(N, U, Term).
bi_ground(L, L, Term) :- !,
     arg(L, Term, Arg),
     bi_ground(Arg).
bi_ground(_, _, _). % L>U: no elements to process
```

Counters: bisection (if-then-else version)

```
bi_ground2(Term) :-
     nonvar(Term),
     functor(Term, _, Arity),
     bi_ground2(1, Arity, Term).

bi_ground2(L, U, Term) :-
     ( L<U ->
         M is (L+U)//2,
         N is M+1,
         bi_ground2(L, M, Term),
         bi_ground2(N, U, Term)
     ;   (L>U -> true
         ;   arg(L, Term, Arg), % L=U
             bi_ground2(Arg)
         )
     ).
```

Counting without numbers

```
num_twice_as_long(L1,L2) :-
     length(L1,N1),
     N2 is N1*2,
     length(L2,N2).


twice_as_long([],[]).
twice_as_long([_|L1],[_,_|L2]) :-
     twice_as_long(L1,L2).
```

Backwards Correctness

Check each clause for:

- When does it make sense to try this clause?
- Does the program ensure that Prolog knows when it doesn't make sense?

Backwards Correctness: A problem case

```
wrong_count_atom_arguments(Term, Count) :-
     nonvar(Term),
     functor(Term, _, Arity),
     wrong_count_atom_arguments(Arity, Term, 0, Count).

wrong_count_atom_arguments(0, _, Count, Count).
wrong_count_atom_arguments(N, Term, Count0, Count) :-
     arg(N, Term, Arg),
     atom(Arg),
     Count1 is Count0+1,
     M is N-1,
     wrong_count_atom_arguments(M, Term, Count1, Count).
wrong_count_atom_arguments(N, Term, Count0, Count) :-
     M is N-1,
     wrong_count_atom_arguments(M, Term, Count0, Count).
```

Backwards Correctness: Problem case eliminated

```
count_atom_arguments(Term, Count) :-
     nonvar(Term),
     functor(Term, _, Arity),
     count_atom_arguments(Arity, Term, 0, Count).

count_atom_arguments(0, _, Count, Count).
count_atom_arguments(N, Term, Count0, Count) :-
     arg(N, Term, Arg),
     ( atom(Arg) ->  Increment = 1  % Arg is atom
     ;               Increment = 0  % Arg is non-atom
     ),
     Count1 is Count0+Increment,
     M is N-1,
     count_atom_arguments(M, Term, Count1, Count).
```

Eliminating one more choice point

```
fast_count_atom_arguments(Term, Count) :-
     nonvar(Term),
     functor(Term, _, Arity),
     fast_count_atom_arguments(Arity, Term, 0, Count).

fast_count_atom_arguments(N, Term, Count0, Count) :-
     ( N=:=0 -> Count is Count0 % no more arguments
     ;          arg(N, Term, Arg),
                ( atom(Arg) ->  Increment = 1  % Arg is atom
                ;               Increment = 0  % Arg is non-atom
                ),
                Count1 is Count0+Increment,
                M is N-1,
                fast_count_atom_arguments(M, Term, Count1, Count)
     ).
```

Unavoidable problems

```
append([], L, L).
append([H|T], L, [H|R]) :-
     append(T,L,R).

| ?- append(X,[],X).

X = [] ? ;
X = [_A] ? ;
X = [_A,_B] ? ;
X = [_A,_B,_C] ? ;
...
```

Since solution space *is* infinite, only possibility is to add comment:

`% append/3: first or third argument must be proper lists`

Comments on practical matters

- Thoroughly read each chapter, also when not presenting.
- Try out the example code.
- Make slides & handouts for when you present and send them to me before Monday morning for comments and inclusion on course page.
- Intermediate results of projects are presented in last class, final results will normally be presented in the next quarter's Clippers

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Various loose ends

- Both Sicstus and SWI Prolog use last-call optimization
- Have people tried the debuggers? (Sicstus in Emacs and graphical SWI, including editing)
- Lexical scoping of variables: Assuming a procedure P declared as part of a procedure Q, the variables visible in P are those declared in P plus those declared in Q.