Remembering subresults (Part I): Well-formed substring tables

Detmar Meurers: Intro to Computational Linguistics I
OSU, LING 684.01, 12. February 2004

Problem: Inefficiency of recomputing subresults

Two example sentences and their potential analysis:

(1) He [gave [the young cat] [to Bill]].
(2) He [gave [the young cat] [some milk]].

The corresponding grammar rules:

```
vp ---> [v_ditrans, np, pp_to].
vp ---> [v_ditrans, np, np].
```

Solution: Memoization

- Store intermediate results:

  a) completely analyzed constituents:
  well-formed substring table or **(passive) chart**

  b) partial and complete analyses:
  (active) chart

- All intermediate results need to be stored for completeness.

- All possible solutions are explored in parallel.

CFG Parsing: The Cocke Younger Kasami Algorithm

- Grammar has to be in Chomsky Normal Form (CNF), only
  - RHS with a single terminal: $A \rightarrow a$
  - RHS with two non-terminals: $A \rightarrow BC$
  - no ϵ rules ($A \rightarrow \epsilon$)

- A representation of the string showing positions and word indices:

$$\cdot_0\ w_1\ \cdot_1\ w_2\ \cdot_2\ w_3\ \cdot_3\ w_4\ \cdot_4\ w_5\ \cdot_5\ w_6\ \cdot_6$$

For example: $\cdot_0$ the $\cdot_1$ young $\cdot_2$ boy $\cdot_3$ saw $\cdot_4$ the $\cdot_5$ dragon $\cdot_6$

The well-formed substring table (= passive chart)

- The well-formed substring table, henceforth (passive) chart, for a string of length n is an $n \times n$ matrix.

- The field (i, j) of the chart encodes the set of all categories of constituents that start at position i and end at position j, i.e.
chart(i,j) $= \{A \mid A \Rightarrow^* \; w_{i+1} \ldots w_j\}$

- The matrix is triangular since no constituent ends before it starts.

Coverage Represented in the Chart

An input sentence with 6 words:

$$\cdot_0\ w_1\ \cdot_1\ w_2\ \cdot_2\ w_3\ \cdot_3\ w_4\ \cdot_4\ w_5\ \cdot_5\ w_6\ \cdot_6$$

Coverage represented in the chart:

TO:

FROM:	1	2	3	4	5	6
0	0–1	0–2	0–3	0–4	0–5	0–6
1		1–2	1–3	1–4	1–5	1–6
2			2–3	2–4	2–5	2–6
3				3–4	3–5	3–6
4					4–5	4–6
5						5–6

Example for Coverage Represented in Chart

Example sentence:

$$\cdot_0 \text{ the } \cdot_1 \text{ young } \cdot_2 \text{ boy } \cdot_3 \text{ saw } \cdot_4 \text{ the } \cdot_5 \text{ dragon } \cdot_6$$

Coverage represented in chart:

	1	2	3	4	5	6
0	the	the young	the young boy	the young boy saw	the young boy saw the	the young boy saw the dragon
1		young	young boy	young boy saw	young boy saw the	young boy saw the dragon
2			boy	boy saw	boy saw the	boy saw the dragon
3				saw	saw the	saw the dragon
4					the	the dragon
5						dragon

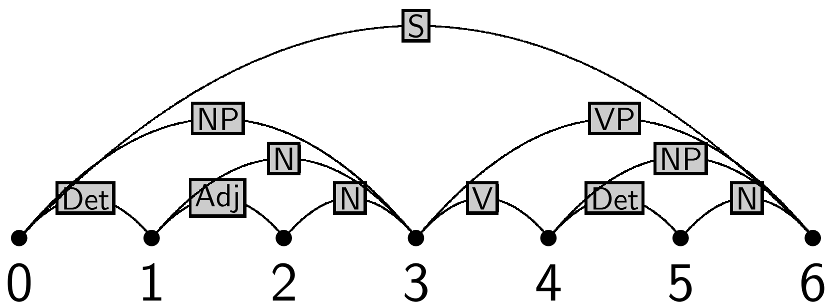
An Example for a Filled-in Chart

Input sentence:

$\cdot_0$ the $\cdot_1$ young $\cdot_2$ boy $\cdot_3$ saw $\cdot_4$ the $\cdot_5$ dragon $\cdot_6$

Chart:

	1	2	3	4	5	6
0	{Det}	{}	{NP}	{}	{}	{S}
1		{Adj}	{N}	{}	{}	{}
2			{N}	{}	{}	{}
3				{V, N}	{}	{VP}
4					{Det}	{NP}
5						{N}



Grammar:

S $\rightarrow$ NP VP
VP $\rightarrow$ Vt NP
NP $\rightarrow$ Det N
N $\rightarrow$ Adj N
Vt $\rightarrow$ saw
Det $\rightarrow$ the
Det $\rightarrow$ a
N $\rightarrow$ dragon
N $\rightarrow$ boy
N $\rightarrow$ saw
Adj $\rightarrow$ young

Filling in the Chart

- It is important to fill in the chart systematically.
- We build all constituents that end at a certain point before we build constituents that end at a later point.

	1	2	3	4	5	6
0						
1						
2						
3						
4						
5						

for j := 1 to length($string$)
    lexical_chart_fill($j-1, j$)
    for $i := j-2$ down to 0
        syntactic_chart_fill(i, j)

Filling in the Chart

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- We build all constituents that end at a certain point before we build constituents that end at a later point.

	1	2	3	4	5	6
0	**1**					
1						
2						
3						
4						
5						

for j := 1 to length($string$)
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  for $i := j-2$ down to 0
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Filling in the Chart

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	1	2	3	4	5	6
0	**1**					
1		**2**				
2						
3						
4						
5						

for j := 1 to length($string$)
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	1	2	3	4	5	6
0	**1**	**3**				
1		**2**				
2						
3						
4						
5						

for j := 1 to length($string$)
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	1	2	3	4	5	6
0	**1**	**3**				
1		**2**				
2			**4**			
3						
4						
5						

for $j := 1$ to length($string$)
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	1	2	3	4	5	6
0	**1**	**3**				
1		**2**	**5**			
2			**4**			
3						
4						
5						

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	1	2	3	4	5	6
0	**1**	**3**	**6**			
1		**2**	**5**			
2			**4**			
3						
4						
5						

for $j := 1$ to length($string$)
    lexical_chart_fill$(j-1, j)$
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Filling in the Chart

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	1	2	3	4	5	6
0	**1**	**3**	**6**			
1		**2**	**5**			
2			**4**			
3				**7**		
4						
5						

for j := 1 to length($string$)
    lexical_chart_fill($j-1, j$)
    for $i := j - 2$ down to 0
        syntactic_chart_fill(i, j)

Filling in the Chart

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	1	2	3	4	5	6
0	**1**	**3**	**6**			
1		**2**	**5**			
2			**4**	**8**		
3				**7**		
4						
5						

for $j := 1$ to length($string$)
    lexical_chart_fill($j - 1, j$)
    for $i := j - 2$ down to 0
        syntactic_chart_fill(i, j)

Filling in the Chart

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	1	2	3	4	5	6
0	**1**	**3**	**6**			
1		**2**	**5**	**9**		
2			**4**	**8**		
3				**7**		
4						
5						

for j := 1 to length($string$)
    lexical_chart_fill($j-1, j$)
    for $i := j-2$ down to 0
        syntactic_chart_fill(i, j)

Filling in the Chart

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- We build all constituents that end at a certain point before we build constituents that end at a later point.

	1	2	3	4	5	6
0	**1**	**3**	**6**	**10**		
1		**2**	**5**	**9**		
2			**4**	**8**		
3				**7**		
4						
5						

for j := 1 to length($string$)
    lexical_chart_fill($j-1, j$)
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	1	2	3	4	5	6
0	**1**	**3**	**6**	**10**		
1		**2**	**5**	**9**		
2			**4**	**8**		
3				**7**		
4					**11**	
5						

for $j := 1$ to length($string$)
    lexical_chart_fill$(j-1, j)$
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	1	2	3	4	5	6
0	**1**	**3**	**6**	**10**		
1		**2**	**5**	**9**		
2			**4**	**8**		
3				**7**	**12**	
4					**11**	
5						

for j := 1 to length($string$)
    lexical_chart_fill($j-1, j$)
    for $i := j-2$ down to 0
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	1	2	3	4	5	6
0	**1**	**3**	**6**	**10**		
1		**2**	**5**	**9**		
2			**4**	**8**	**13**	
3				**7**	**12**	
4					**11**	
5						

for j := 1 to length($string$)
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Filling in the Chart

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	1	2	3	4	5	6
0	**1**	**3**	**6**	**10**		
1		**2**	**5**	**9**	**14**	
2			**4**	**8**	**13**	
3				**7**	**12**	
4					**11**	
5						

for j := 1 to length($string$)
  lexical_chart_fill($j-1, j$)
  for $i := j-2$ down to 0
    syntactic_chart_fill(i, j)

Filling in the Chart

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	1	2	3	4	5	6
0	**1**	**3**	**6**	**10**	**15**	
1		**2**	**5**	**9**	**14**	
2			**4**	**8**	**13**	
3				**7**	**12**	
4					**11**	
5						

for j := 1 to length($string$)
    lexical_chart_fill$(j-1, j)$
    for $i := j-2$ down to 0
        syntactic_chart_fill(i, j)

Filling in the Chart

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	1	2	3	4	5	6
0	**1**	**3**	**6**	**10**	**15**	
1		**2**	**5**	**9**	**14**	
2			**4**	**8**	**13**	
3				**7**	**12**	
4					**11**	
5						**16**

for j := 1 to length($string$)
    lexical_chart_fill($j-1, j$)
    for $i := j-2$ down to 0
        syntactic_chart_fill(i, j)

Filling in the Chart

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- We build all constituents that end at a certain point before we build constituents that end at a later point.

	1	2	3	4	5	6
0	**1**	**3**	**6**	**10**	**15**	
1		**2**	**5**	**9**	**14**	
2			**4**	**8**	**13**	
3				**7**	**12**	
4					**11**	**17**
5						**16**

for $j := 1$ to length($string$)
    lexical_chart_fill$(j-1, j)$
    for $i := j-2$ down to 0
        syntactic_chart_fill(i, j)

Filling in the Chart

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- We build all constituents that end at a certain point before we build constituents that end at a later point.

	1	2	3	4	5	6
0	**1**	**3**	**6**	**10**	**15**	
1		**2**	**5**	**9**	**14**	
2			**4**	**8**	**13**	
3				**7**	**12**	**18**
4					**11**	**17**
5						**16**

for j := 1 to length($string$)
    lexical_chart_fill($j-1, j$)
    for i := $j-2$ down to 0
        syntactic_chart_fill(i, j)

Filling in the Chart

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	1	2	3	4	5	6
0	**1**	**3**	**6**	**10**	**15**	
1		**2**	**5**	**9**	**14**	
2			**4**	**8**	**13**	**19**
3				**7**	**12**	**18**
4					**11**	**17**
5						**16**

for $j := 1$ to length($string$)
  lexical_chart_fill($j-1, j$)
  for $i := j-2$ down to 0
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Filling in the Chart

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	1	2	3	4	5	6
0	**1**	**3**	**6**	**10**	**15**	
1		**2**	**5**	**9**	**14**	**20**
2			**4**	**8**	**13**	**19**
3				**7**	**12**	**18**
4					**11**	**17**
5						**16**

for $j := 1$ to length($string$)
    lexical_chart_fill($j-1, j$)
    for $i := j-2$ down to 0
        syntactic_chart_fill(i, j)

Filling in the Chart

- It is important to fill in the chart systematically.
- We build all constituents that end at a certain point before we build constituents that end at a later point.

	1	2	3	4	5	6
0	**1**	**3**	**6**	**10**	**15**	**21**
1		**2**	**5**	**9**	**14**	**20**
2			**4**	**8**	**13**	**19**
3				**7**	**12**	**18**
4					**11**	**17**
5						**16**

for $j := 1$ to length($string$)
    lexical_chart_fill$(j-1, j)$
    for $i := j-2$ down to 0
        syntactic_chart_fill(i, j)

lexical_chart_fill(j-1,j)

- Idea: Lexical lookup. Fill the field $(j-1, j)$ in the chart with the preterminal category dominating word j.
- Realized as:

$$chart(j-1, j) := \{\mathsf{X} \mid \mathsf{X} \rightarrow \mathsf{word}_j \in \mathsf{P}\}$$

syntactic_chart_fill(i,j)

- Idea: Perform all reduction step using syntactic rules such that the reduced symbol covers the string from i to j.

- Realized as: $chart(i,j) = \left\{ A \;\middle|\; \begin{array}{l} A \rightarrow BC \in P, \\ i < k < j, \\ B \in chart(i,k), \\ C \in chart(k,j) \end{array} \right\}$

- Explicit loops over every possible value of k and every context free rule:

  $chart(i,j) := \{\}$.
  for $k := i + 1$ to $j - 1$
    for every $A \rightarrow BC \in P$
      if $B \in chart(i,k)$ and $C \in chart(k,j)$ then
        $chart(i,j) := chart(i,j) \cup \{\text{A}\}$.

The Complete CYK Algorithm

Input: start category S and input $string$

$n :=$ length($string$)

for $j := 1$ to n

  $chart(j-1, j) := \{\mathsf{X} \mid \mathsf{X} \rightarrow \mathsf{word}_j \in \mathsf{P}\}$

  for $i := j-2$ down to 0

    $chart(i, j) := \{\}$

    for $k := i+1$ to $j-1$

      for every $A \rightarrow BC \in P$

        if $B \in chart(i, k)$ and $C \in chart(k, j)$ then

          $chart(i, j) := chart(i, j) \cup \{\mathsf{A}\}$

Output: if $S \in chart(0, n)$ then accept else reject

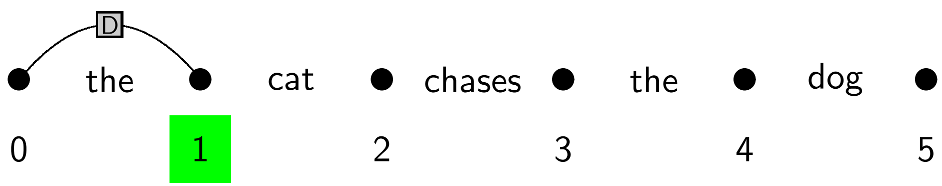
Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

Lexical Entry: *the* ($j = 1$, field chart(0,1))

	1	2	3	4	5
0	d				
1					
2					
3					
4					



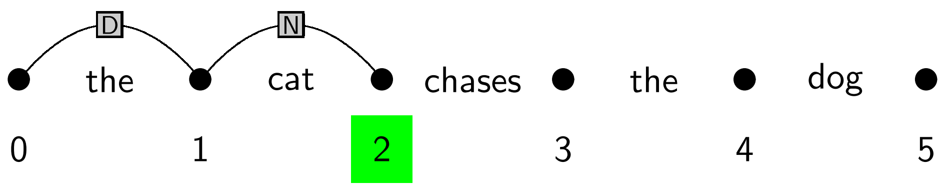
Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

Lexical Entry: *cat* (j = 2 , field chart(1,2))

	1	2	3	4	5
0	d				
1		n			
2					
3					
4					



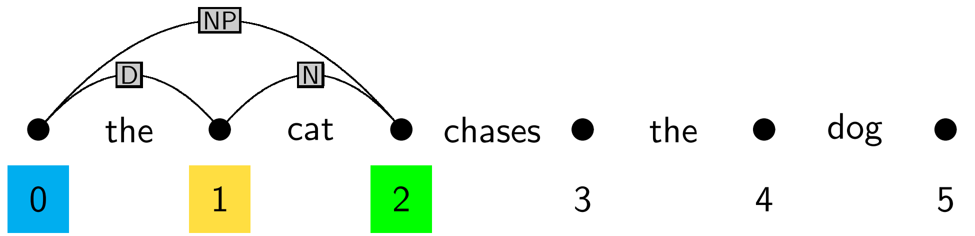
Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

j = 2
i = 0
k = 1

	1	2	3	4	5
0	d	np			
1		n			
2					
3					
4					



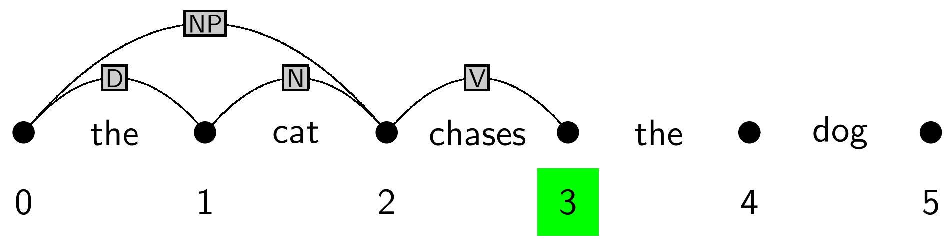
Example Application of the CYK Algorithm

s → np vp
np → d n
vp → v np

d → the
n → dog
n → cat
v → chases

Lexical Entry: *chases* ($j = 3$, field chart(2,3))

	1	2	3	4	5
0	d	np			
1		n			
2			v		
3					
4					



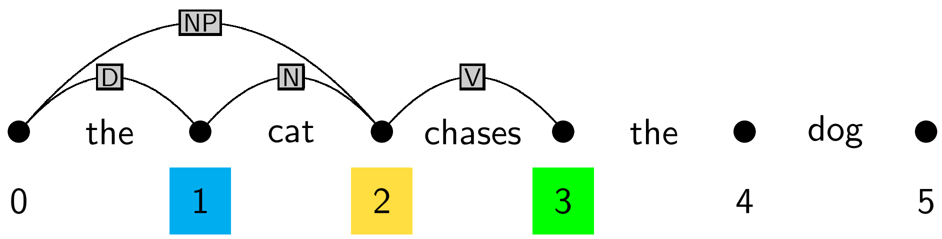
Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

$j = 3$
$i = 1$
$k = 2$

	1	2	3	4	5
0	d	np			
1		n			
2			v		
3					
4					



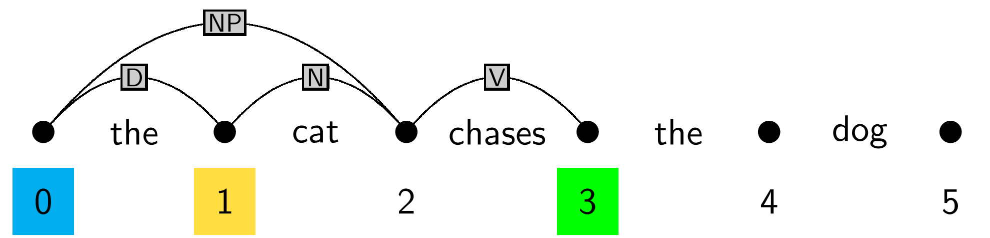
Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

j = 3
i = 0
k = 1

	1	2	3	4	5
0	$\boxed{\text{d}}$	np	$\boxed{}$		
1		n	$\boxed{}$		
2			v		
3					
4					



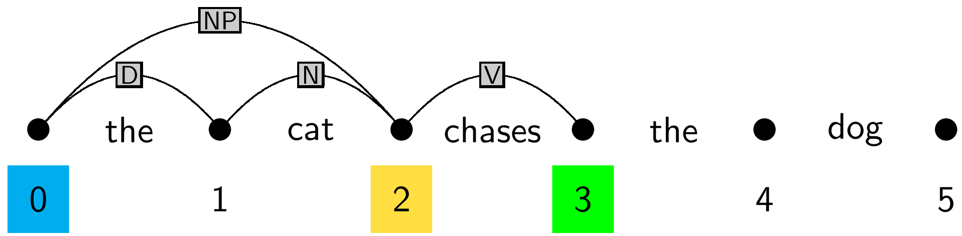
Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

$j = 3$
$i = 0$
$k = 2$

	1	2	3	4	5
0	d	np			
1		n			
2			v		
3					
4					



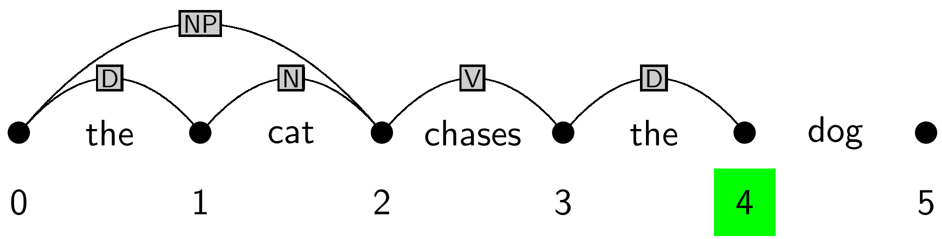
Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

Lexical Entry: *the* (j = 4 , field chart(3,4))

	1	2	3	4	5
0	d	np			
1		n			
2			v		
3				$\boxed{\text{d}}$	
4					



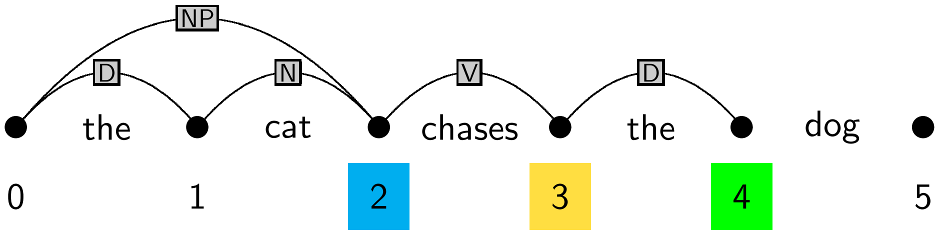
Example Application of the CYK Algorithm

s → np vp
np → d n
vp → v np

d → the
n → dog
n → cat
v → chases

$j = 4$
$i = 2$
$k = 3$

	1	2	3	4	5
0	d	np			
1		n			
2			v		
3				d	
4					



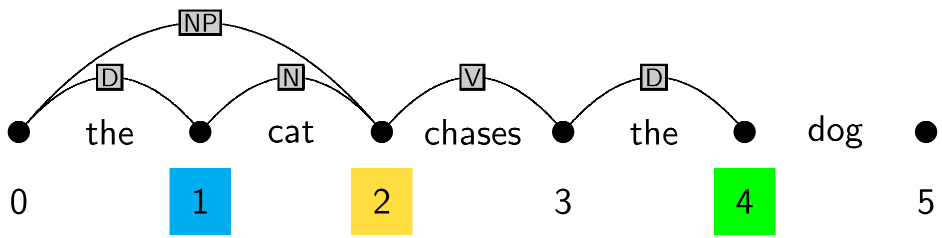
Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

j = 4
i = 1
k = 2

	1	2	3	4	5
0	d	np			
1		n			
2			v		
3				d	
4					

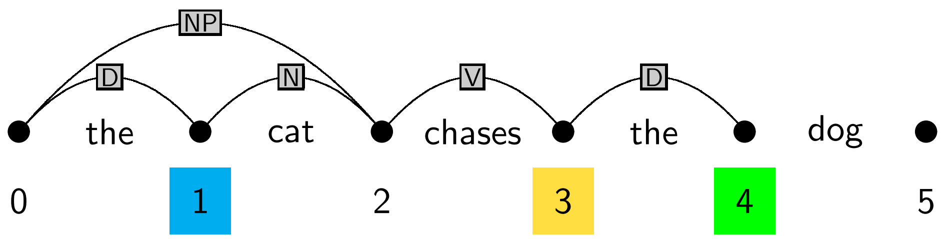


Example Application of the CYK Algorithm

$s \longrightarrow np\ vp$ $\qquad d \longrightarrow the$
$np \longrightarrow d\ n$ $\qquad n \longrightarrow dog$
$vp \longrightarrow v\ np$ $\qquad n \longrightarrow cat$
$\qquad v \longrightarrow chases$

$j = 4$
$i = 1$
$k = 3$

	1	2	3	4	5
0	d	np			
1		n	$\boxed{}$	$\boxed{}$	
2			v		
3				$\boxed{d}$	
4					

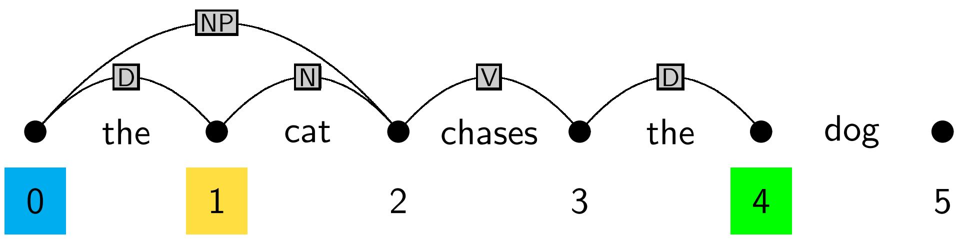


Example Application of the CYK Algorithm

s $\longrightarrow$ np vp	d $\longrightarrow$ the
np $\longrightarrow$ d n	n $\longrightarrow$ dog
vp $\longrightarrow$ v np	n $\longrightarrow$ cat
	v $\longrightarrow$ chases

j = 4
i = 0
k = 1

	1	2	3	4	5
0	$\boxed{\text{d}}$	np		$\boxed{}$	
1		n		$\boxed{}$	
2			v		
3				d	
4					

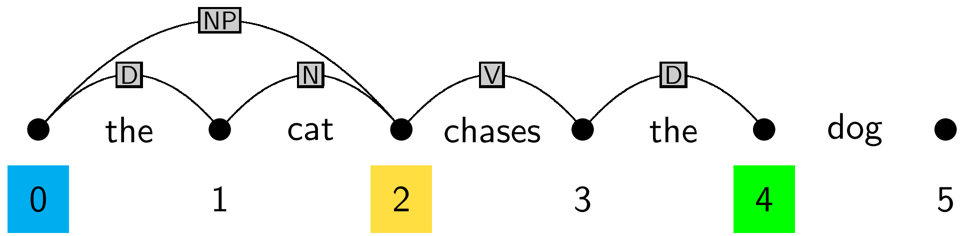


Example Application of the CYK Algorithm

s → np vp	d → the
np → d n	n → dog
vp → v np	n → cat
	v → chases

j = 4
i = 0
k = 2

	1	2	3	4	5
0	d	$\boxed{\text{np}}$		$\boxed{}$	
1		n			
2			v	$\boxed{}$	
3				d	
4					



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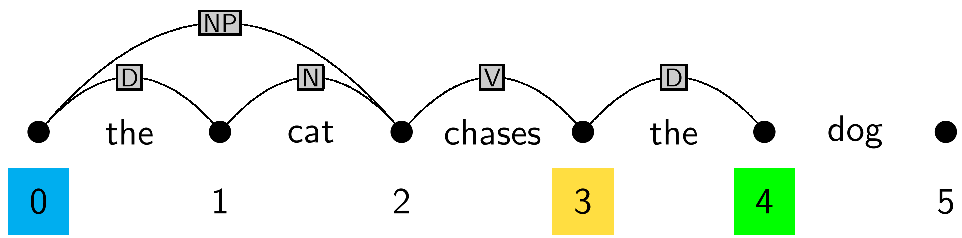
0	d	np	$\boxed{}$	$\boxed{}$	
1		n			
2			v		
3				$\boxed{d}$	
4					



Example Application of the CYK Algorithm

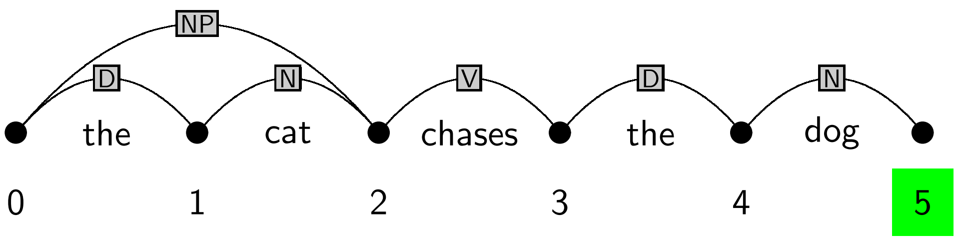
s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

Lexical Entry: *dog*

($j = 5$, field chart(4,5))

	1	2	3	4	5
0	d	np			
1		n			
2			v		
3				d	
4					$\boxed{\text{n}}$



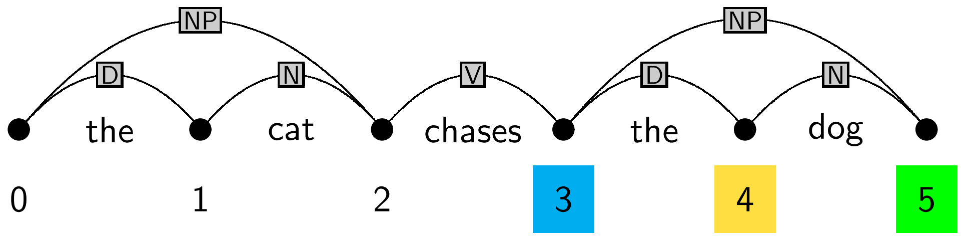
Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

j = 5
i = 3
k = 4

	1	2	3	4	5
0	d	np			
1		n			
2			v		
3				$\boxed{\text{d}}$	$\boxed{\text{np}}$
4					$\boxed{\text{n}}$



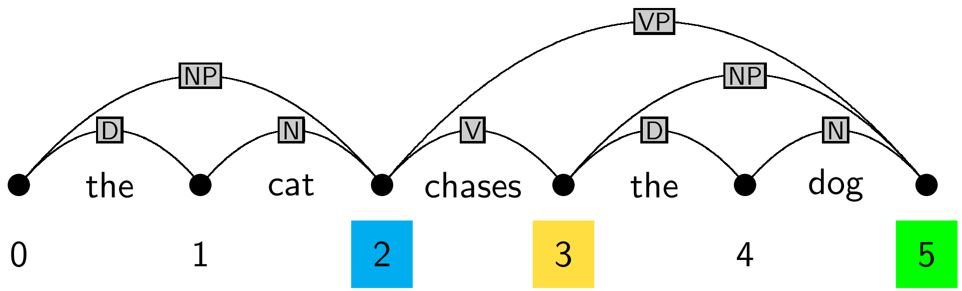
Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

j = 5
i = 2
k = 3

	1	2	3	4	5
0	d	np			
1		n			
2			v		vp
3				d	np
4					n



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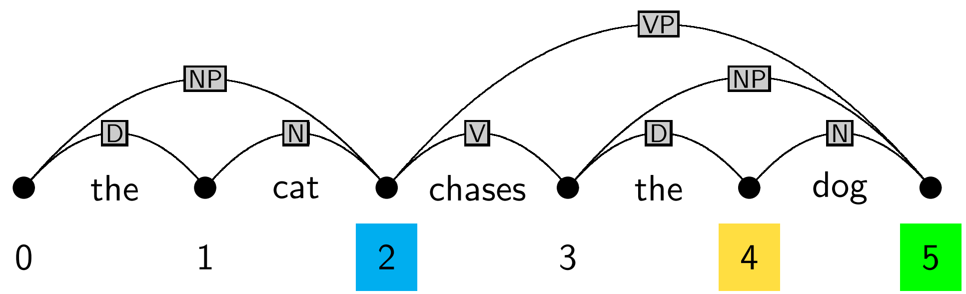
$i = 2$
$k = 4$

	1	2	3	4	5
0	d	np			
1		n			
2			v	$\boxed{}$	$\boxed{\text{vp}}$
3				d	np
4					$\boxed{\text{n}}$



Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

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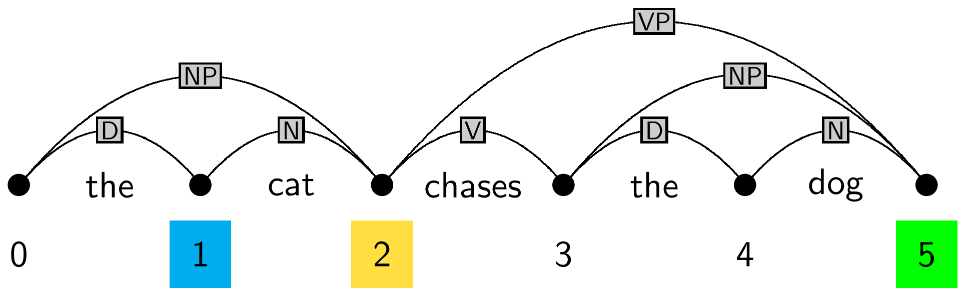
i = 1
k = 2

	1	2	3	4	5
0	d	np			
1		n			
2			v		vp
3				d	np
4					n



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np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
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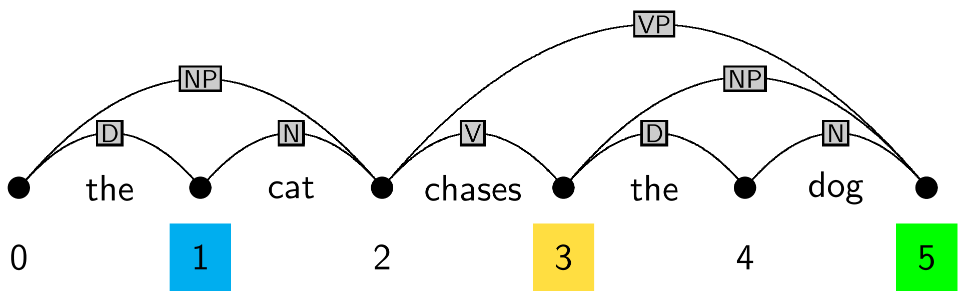
	1	2	3	4	5
0	d	np			
1		n	$\boxed{}$		$\boxed{}$
2			v		vp
3				d	$\boxed{\text{np}}$
4					n



Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
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v $\longrightarrow$ chases

j = 5
i = 1
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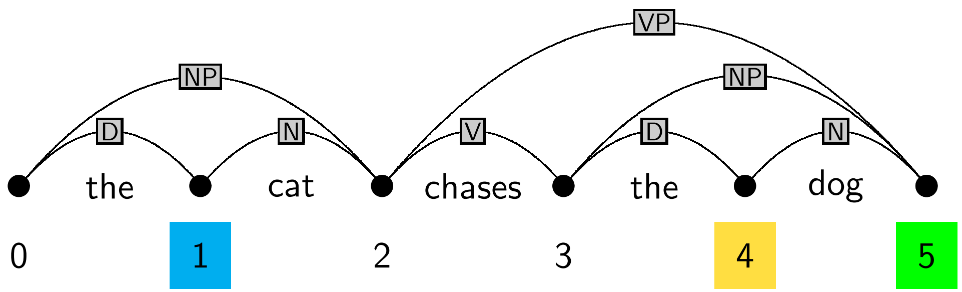
	1	2	3	4	5
0	d	np			
1		n		$\boxed{}$	$\boxed{}$
2			v		vp
3				d	np
4					$\boxed{n}$



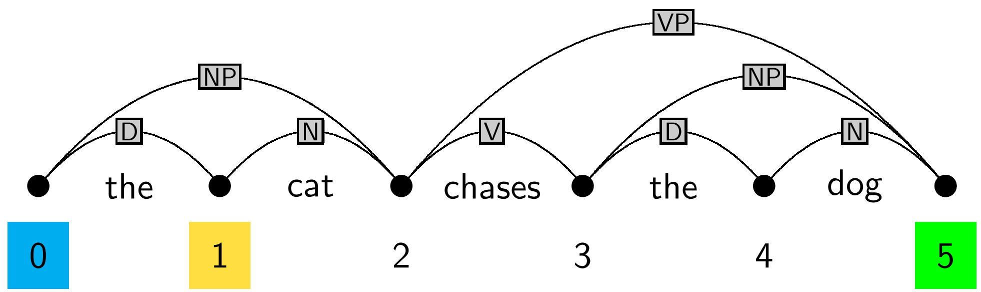
Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

$j = 5$
$i = 0$
$k = 1$

	1	2	3	4	5
0	$\boxed{\text{d}}$	np			$\boxed{}$
1		n			$\boxed{}$
2			v		vp
3				d	np
4					n



Example Application of the CYK Algorithm

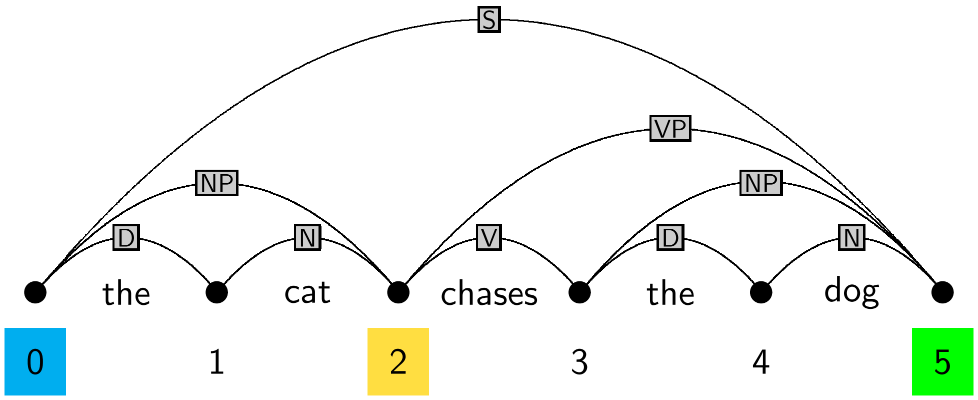
$s \longrightarrow np\ vp$	$d \longrightarrow$ the
$np \longrightarrow d\ n$	$n \longrightarrow$ dog
$vp \longrightarrow v\ np$	$n \longrightarrow$ cat
	$v \longrightarrow$ chases

$j = 5$

$i = 0$

$k = 2$

	1	2	3	4	5
0	d	$\boxed{np}$			$\boxed{s}$
1		n			
2			v		$\boxed{vp}$
3				d	np
4					n



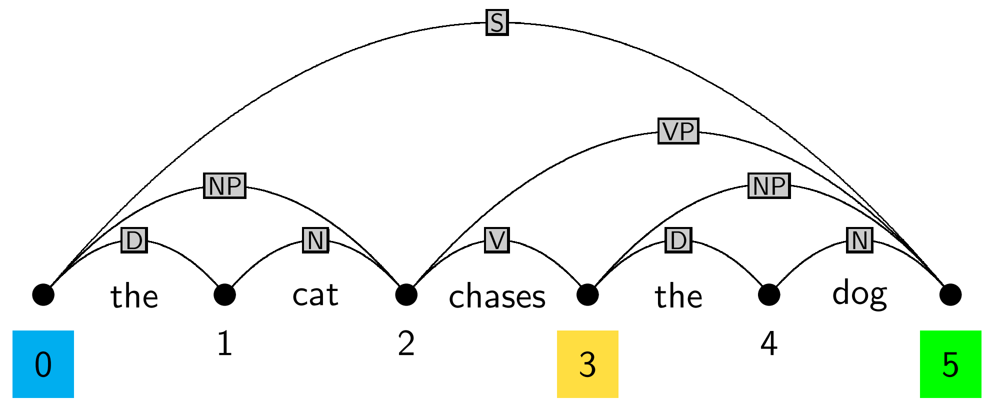
Example Application of the CYK Algorithm

s $\longrightarrow$ np vp
np $\longrightarrow$ d n
vp $\longrightarrow$ v np

d $\longrightarrow$ the
n $\longrightarrow$ dog
n $\longrightarrow$ cat
v $\longrightarrow$ chases

$j = 5$
$i = 0$
$k = 3$

	1	2	3	4	5
0	d	np	$\boxed{}$		$\boxed{\text{s}}$
1		n			
2			v		vp
3				d	$\boxed{\text{np}}$
4					n



Example Application of the CYK Algorithm

s $\longrightarrow$ np vp	d $\longrightarrow$ the
np $\longrightarrow$ d n	n $\longrightarrow$ dog
vp $\longrightarrow$ v np	n $\longrightarrow$ cat
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$j = 5$

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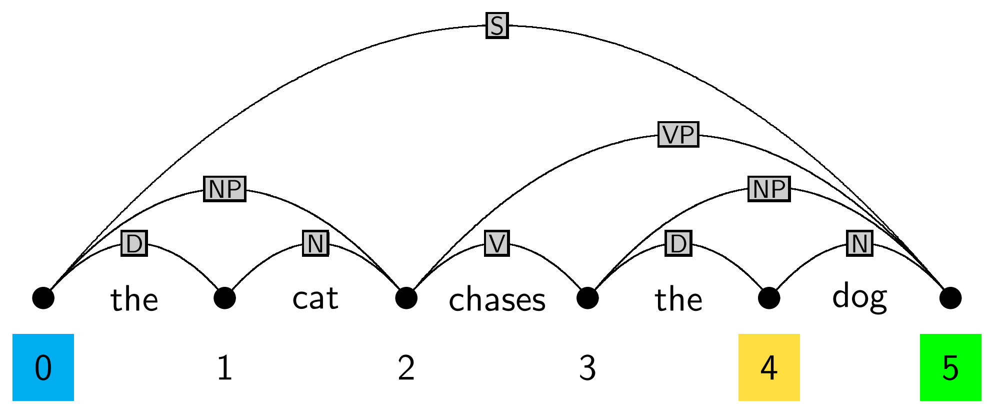
	1	2	3	4	5
0	d	np		$\boxed{}$	$\boxed{\text{s}}$
1		n			
2			v		vp
3				d	np
4					$\boxed{\text{n}}$



Dynamic knowledge bases in PROLOG

- Declaration of a dynamic predicate: `dynamic/1` declaration, e.g:

  ```
  :- dynamic chart/3.
  ```

  to store facts of the form `chart(From,To,Category)`:

- Add a fact to the database: `assert/1`, e.g.:

  ```
  assert(chart(1,3,np)).
  ```

  Special versions `asserta/1`/`assertz/1` ensure adding facts first/last.

- Removing a fact from the database: `retract/1`, e.g.:

  ```
  retract(chart(1,_,np)).
  ```

  To remove all matching facts from the database use `retractall/1`

The CYK algorithm in PROLOG (parser/cky/cky.pl)

```
:- dynamic chart/3.                  % chart(From,To,Category)
:- op(1100,xfx,'--->').              % Operator for grammar rules


% recognize(+WordList,?Startsymbol): top-level of CYK recognizer

recognize(String,Cat) :-
   retractall(chart(_,_,_)),         % initialize chart
   length(String,N),                 % determine length of string
   fill_chart(String,0,N),           % call parser to fill the chart
   chart(0,N,Cat).                   % check whether parse successful
```

```
% fill_chart(+WordList,+Current minus one,+Last)
% J-LOOP from 1 to n

fill_chart([],N,N).
fill_chart([W|Ws],JminOne,N) :-
   J is JminOne + 1,
   lexical_chart_fill(W,JminOne,J),
   %
   I is J - 2,
   syntactic_chart_fill(I,J),
   %
   fill_chart(Ws,J,N).
```

```
% lexical_chart_fill(+Word,+JminOne,+J)
% fill diagonal with preterminals

lexical_chart_fill(W,JminOne,J) :-
   (Cat ---> [W]),
   add_to_chart(JminOne,J,Cat),
   fail
 ; true.
```

```
% syntactic_chart_fill(+I,+J)
% I-LOOP from J-2 downto 0

syntactic_chart_fill(-1,_) :- !.
syntactic_chart_fill(I,J) :-
   K is I+1,
   build_phrases_from_to(I,K,J),
   %
   IminOne is I-1,
   syntactic_chart_fill(IminOne,J).
```

```
% build_phrases_from_to(+I,+Current-K,+J)
% K-LOOP from I+1 to J-1

build_phrases_from_to(_,J,J) :- !.
build_phrases_from_to(I,K,J) :-
   chart(I,K,B),
   chart(K,J,C),
   (A ---> [B,C]),
   add_to_chart(I,J,A),
   fail
  ; KplusOne is K+1,
   build_phrases_from_to(I,KplusOne,J).
```

```
% add_to_chart(+Cat,+From,+To): add if not yet there
add_to_chart(From,To,Cat) :-
   chart(From,To,Cat),
   !.
add_to_chart(From,To,Cat) :-
   assertz(chart(From,To,Cat)).
```