# Introduction to Parsing

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OSU, LING 684.01

# Overview

- What is a parser?
- Under what criteria can they be evaluated?
- Parsing strategies
  - top-down vs. bottom-up
  - left-right vs. right-left
  - depth-first vs. breadth-first
- Implementing different types of parsers:
  - Basic top-down and bottom-up
  - More efficient algorithms

# Parsers and criteria to evaluate them

- Function of a parser:
  - grammar $+$ string $\rightarrow$ analysis trees
- Main criteria for evaluating parsers:
  - correctness
  - completeness
  - efficiency

# Correctness

A parser is **correct** iff for every grammar and for every string, every analysis returned by parser is an actual analysis.

Correctness is nearly always required (unless simple post-processor could eliminate wrong analyses)

# Completeness

A parser is **complete** iff for every grammar and for every string, every correct analysis is found by the parser.

- In theory, always desirable.
- In practice, essential to find the 'relevant' analysis first (possibly using heuristics).
- For grammars licensing an infinite number of analyses this means: there is no analysis that the parser could not find.

# Efficiency

- One can reason about complexity of (parsing) algorithms by considering how it will deal with bigger and bigger examples.

- For practical purposes, the factors ignored by such analyses are at least as important.
  - profiling using typical examples important
  - finding the (relevant) first parse vs. all parse

- Memoization of complete or partial results is essential to obtain efficient parsing algorithms.

# Complexity classes

If $n$ is the length of the string to be parsed, one can distinguish the following complexity classes:

- **constant**: amount of work does not depend on $n$
- **logarithmic**: amount of work behaves like $log_k(n)$ for some constant $k$
- **polynomial**: amount of work behaves like $n^k$, for some constant $k$. This is sometimes subdivided into the cases
  - **linear** $(k = 1)$
  - **quadratic** $(k = 2)$
  - **cubic** $(k = 3)$
  - ...
- **exponential**: amount of work behaves like $k^n$, for some constant $k$.

# Complexity and the Chomsky hierarchy

| Grammar type | Worst-case complexity of recognition |
|---|---|
| regular (3) | linear |
| context-free (2) | cubic ($n^3$) |
| context-sensitive (1) | exponential |
| general rewrite (0) | undecidable |

Recognition with type 0 grammars is **recursively enumerable**: if a string $x$ is in the language, the recognition algorithm will succeed, but it will not return if $x$ is not in the language.

# Parsing strategies

1. What do we start from?
   - top-down vs. bottom-up
2. In what order is the string or the RHS of a rule looked at?
   - left-to-right, right-to-left, island-driven, . . .
3. How are alternatives explored?
   - depth-first vs. breadth-first

# Direction of processing I: Top-down

**Goal-driven** processing is Top-down:

- Start with the start symbol
- Derive sentential forms.
- If the string is among the sentences derived this way, it is part of the language.

# Direction of processing II: Bottom-up

**Data-driven** processing is Bottom-up:

- Start with the sentence.
- For each substring $\sigma$ of each sentential form $\alpha\sigma\beta$, find each grammar rule $N \rightarrow \sigma$ to obtain all sentential forms $\alpha N \beta$.
- If the start symbol is among the sentential forms obtained, the sentence is part of the language.

Problem: Epsilon rules ($N \rightarrow \epsilon$).

# The order in which one looks at a RHS

Left-to-Right

- Use the leftmost symbol first, continuing with the next to its right

## How are alternatives explored? I. Depth-first

- At every choice point: Pursue a single alternative completely before trying another alternative.
- State of affairs at the choice points needs to be remembered. Choices can be discarded after unsuccessful exploration.
- Depth-first search is not necessarily complete.

Problem for top-down, left-to-right, depth-first processing:

- left-recursion
  For example, a rule like N' $\rightarrow$ N' PP leads to non-termination.

# How are alternatives explored? II. Breadth-first

- At every choice point: Pursue every alternative for one step at a time.
- Requires serious bookkeeping since each alternative computation needs to be remembered at the same time.
- Search is guaranteed to be complete.

# Compiling and executing DCGs in Prolog

- DCGs are a grammar formalism supporting any kind of parsing regime.
- The standard translation of DCGs to Prolog plus the proof procedure of Prolog results in a parsing strategy which is
  - top-down
  - left-to-right
  - depth-first

# Implementing parsers

- Data structures: a parser configuration
- Top-down parsing
  - formal characterization
  - Prolog implementation
- Bottom-up parsing
  - formal characterization
  - Prolog implementation
- Towards more efficient parsers:
  - Left-corner
  - Remembering subresults

# An example grammar (parser/simple/grammar.pl)

```
% defining grammar rule operator
:- op(1100,xfx,'--->').

% lexicon:
vt  ---> [saw].
det ---> [the].
det ---> [a].
n   ---> [dragon].
n   ---> [boy].
adj ---> [young].

% syntactic rules:
s  ---> [np, vp].
vp ---> [vt, np].
np ---> [det, n].
n  ---> [adj, n].
```

# A parser configuration

Assuming a left-to-right order of processing, a **configuration** of a parser can be encoded by a pair of

- a stack as auxiliary memory
- the string remaining to be recognized

More formally, for a grammar $G = (N, \Sigma, S, P)$, a parser configuration is a pair $< \alpha, \tau >$ with $\alpha \in (N \cup \Sigma)^*$ and $\tau \in \Sigma^*$

# Top-down parsing

- **Start configuration** for recognizing a string $\omega$: $< S, \omega >$
- **Available actions**:
  - **consume**: remove an expected terminal $a$ from the string
    $< a\alpha, a\tau > \mapsto < \alpha, \tau >$
  - **expand**: apply a phrase structure rule
    $< A\beta, \tau > \mapsto < \alpha\beta, \tau >$ if $A \rightarrow \alpha \ \in P$
- **Success configuration**: $< \epsilon, \epsilon >$

# A top-down parser in Prolog (parser/simple/td_parser.pl)

```
:- op(1100,xfx,'--->').

% Start
td_parse(String) :- td_parse([s],String).

% Success
td_parse([],[]).

% Consume
td_parse([H|T],[H|R]) :-
        td_parse(T,R).

% Expand
td_parse([A|Beta],String) :-
        (A ---> Alpha),
        append(Alpha,Beta,Stack),
        td_parse(Stack,String).
```

# Top-Down, left-right, depth-first tree traversal

$S_1$

$S \rightarrow NP\ VP$
$VP \rightarrow Vt\ NP$
$NP \rightarrow Det\ N$
$N \rightarrow Adj\ N$

$Vt \rightarrow$ saw
$Det \rightarrow$ the
$Det \rightarrow$ a
$N \rightarrow$ dragon
$N \rightarrow$ boy
$Adj \rightarrow$ young

# Top-Down, left-right, depth-first tree traversal

S$_1$

NP$_2$

S $\longrightarrow$ NP VP
VP $\longrightarrow$ Vt NP
NP $\longrightarrow$ Det N
N $\longrightarrow$ Adj N

Vt $\longrightarrow$ saw
Det $\longrightarrow$ the
Det $\longrightarrow$ a
N $\longrightarrow$ dragon
N $\longrightarrow$ boy
Adj $\longrightarrow$ young

# Top-Down, left-right, depth-first tree traversal

S$_1$

NP$_2$

Det$_3$

S $\longrightarrow$ NP VP
VP $\longrightarrow$ Vt NP
NP $\longrightarrow$ Det N
N $\longrightarrow$ Adj N

Vt $\longrightarrow$ saw
Det $\longrightarrow$ the
Det $\longrightarrow$ a
N $\longrightarrow$ dragon
N $\longrightarrow$ boy
Adj $\longrightarrow$ young

# Top-Down, left-right, depth-first tree traversal

$S_1$

$NP_2$

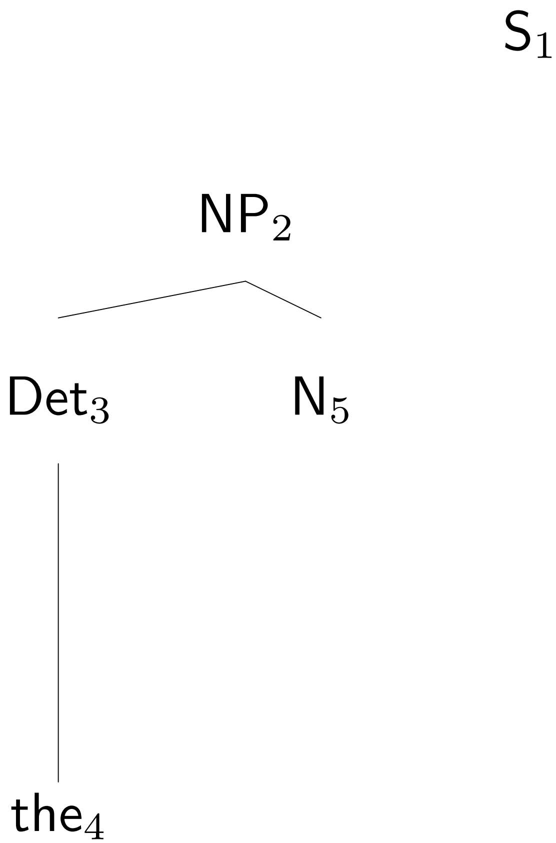
$Det_3$

$the_4$

S → NP VP
VP → Vt NP
NP → Det N
N → Adj N

Vt → saw
Det → the
Det → a
N → dragon
N → boy
Adj → young

# Top-Down, left-right, depth-first tree traversal

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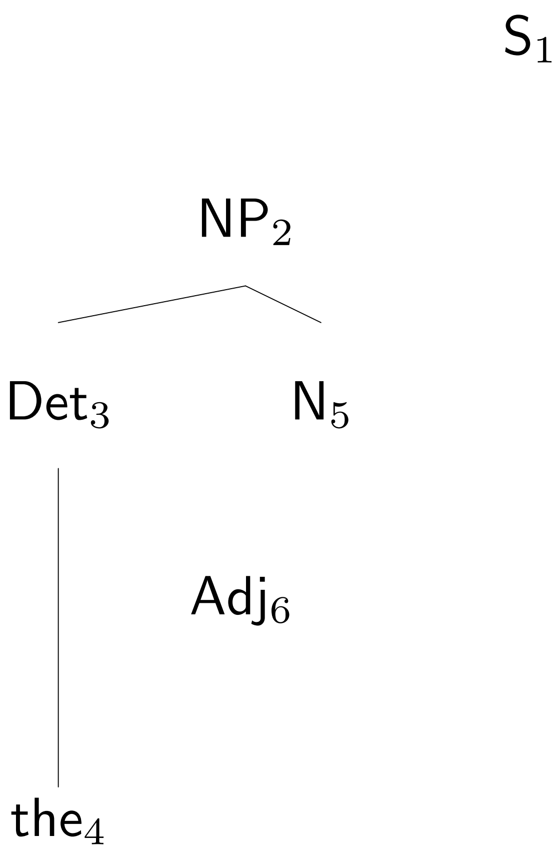


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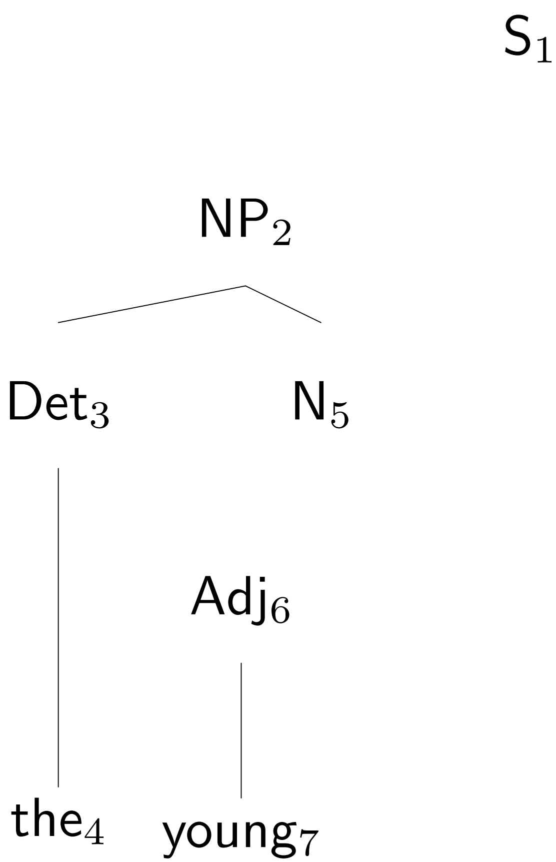


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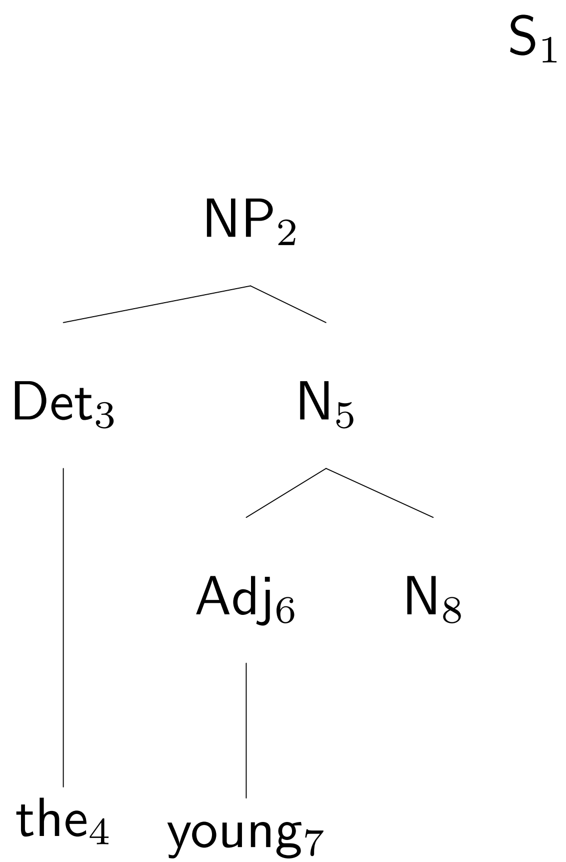


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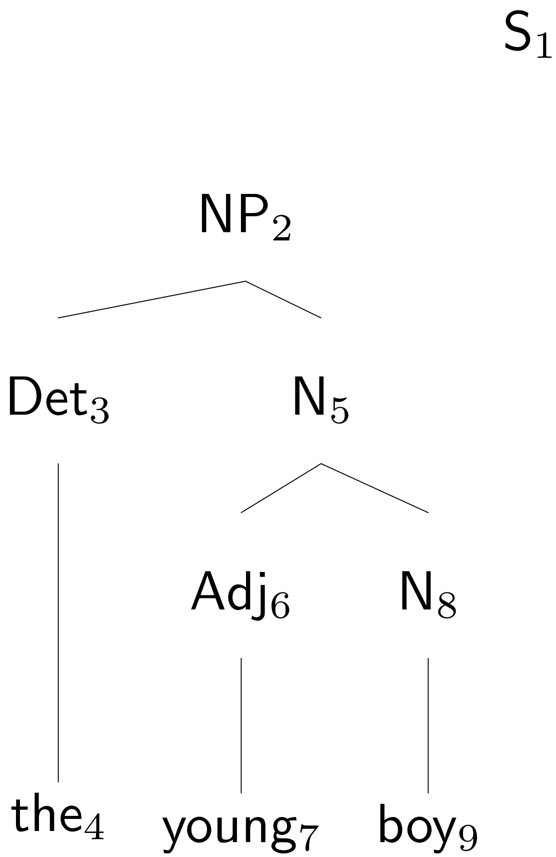


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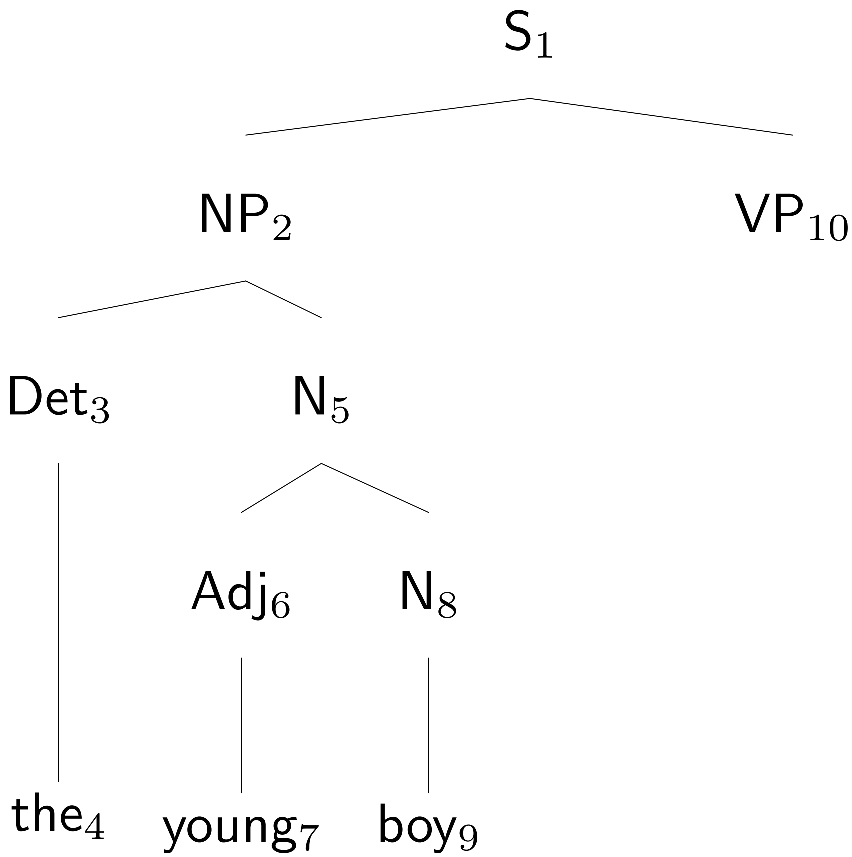


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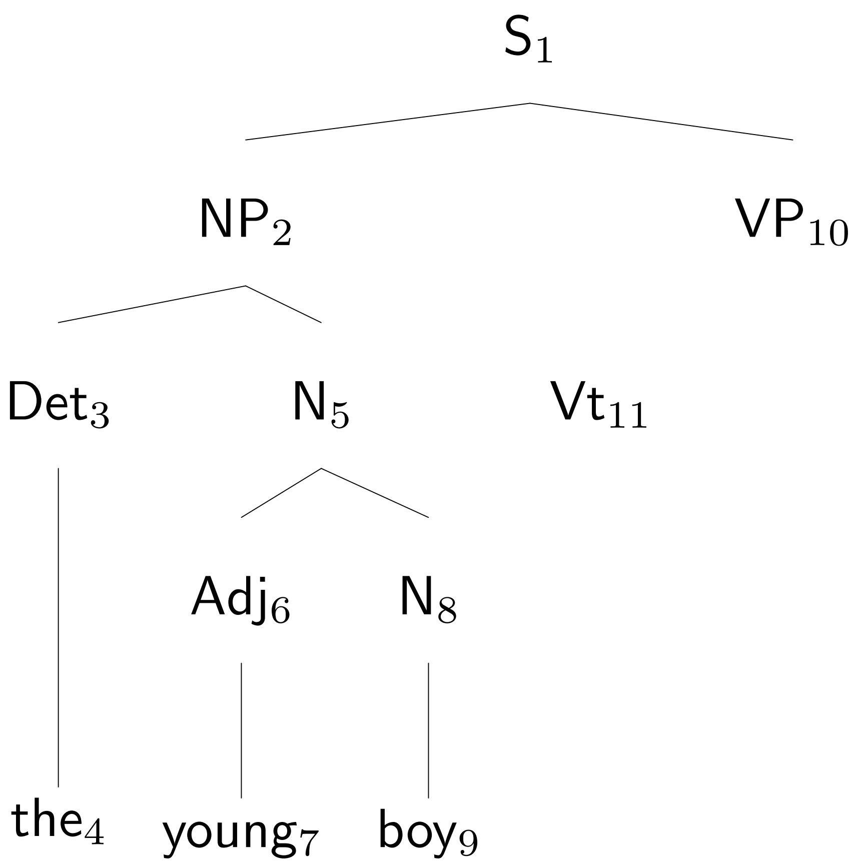


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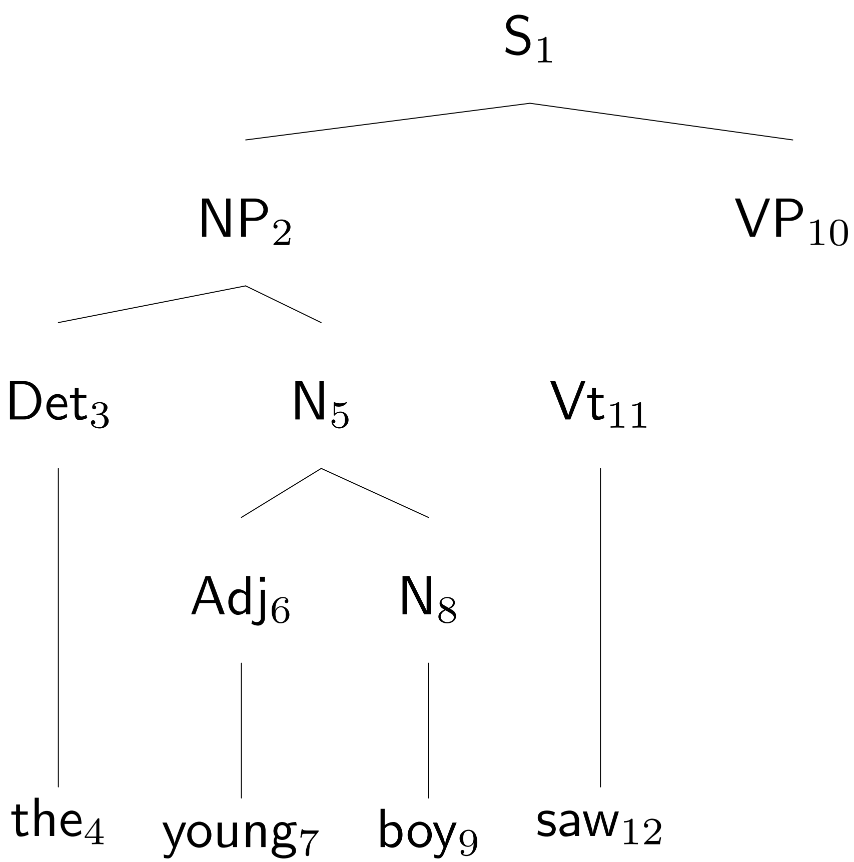


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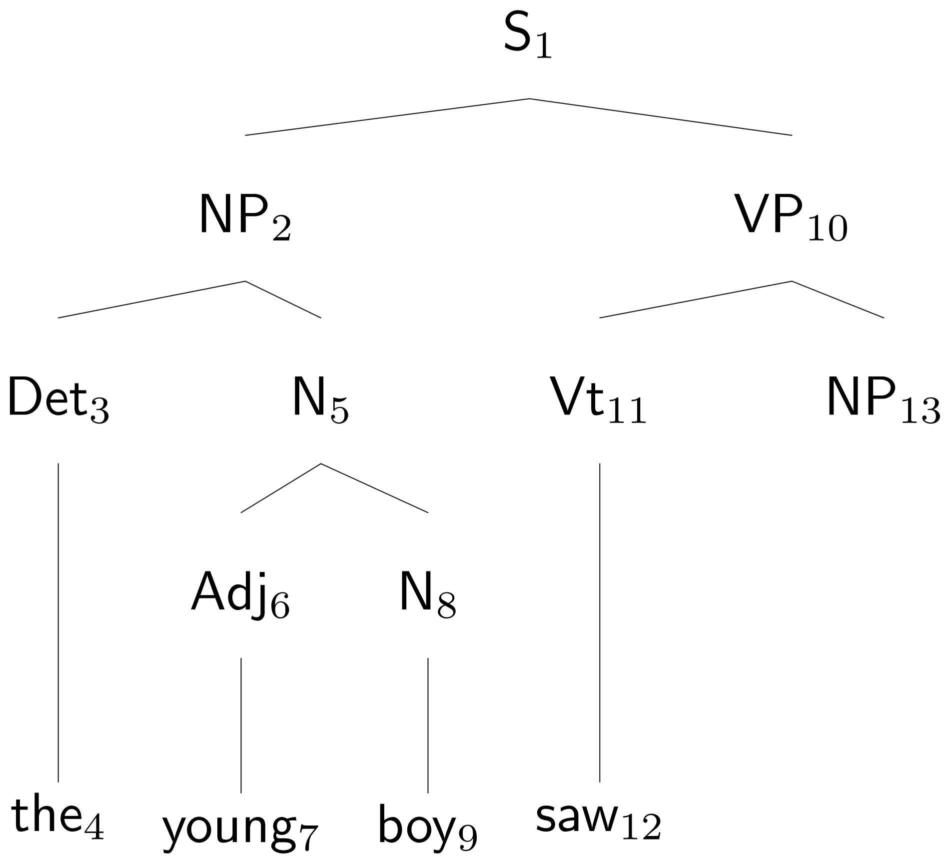


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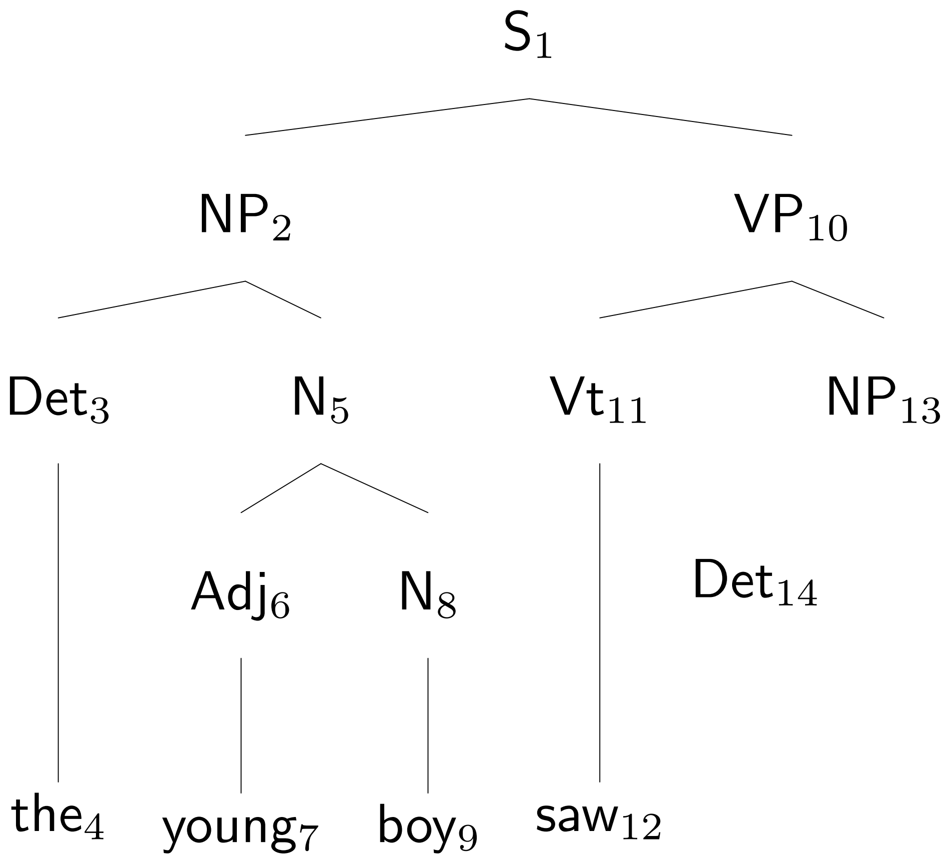


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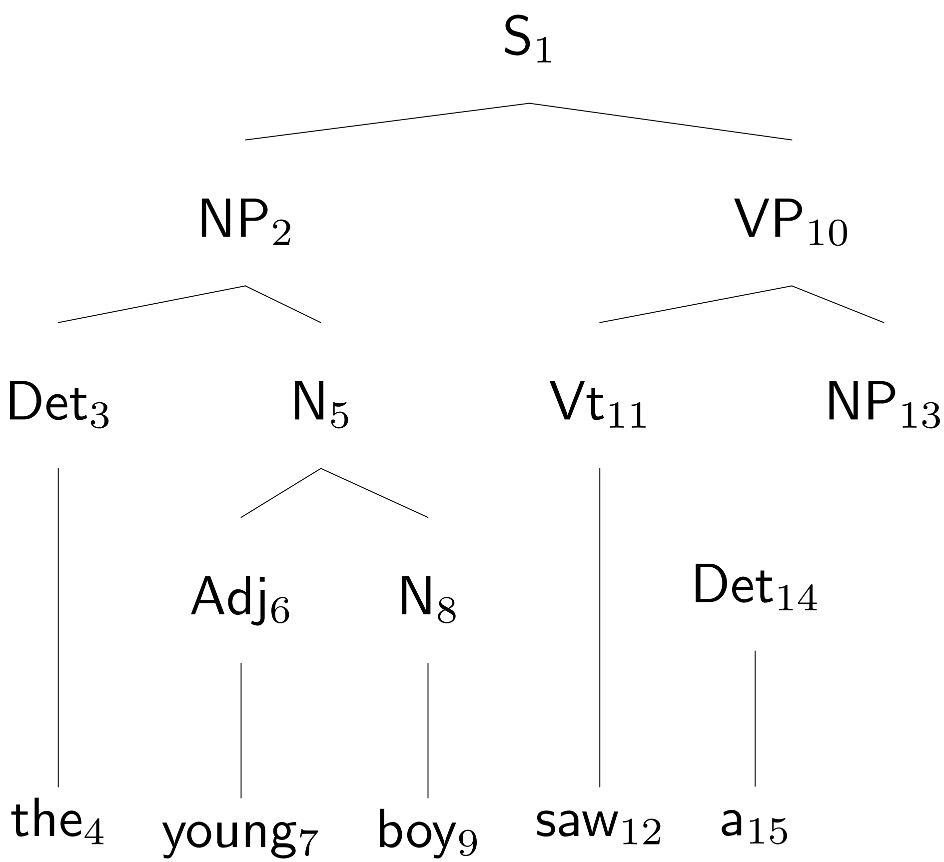


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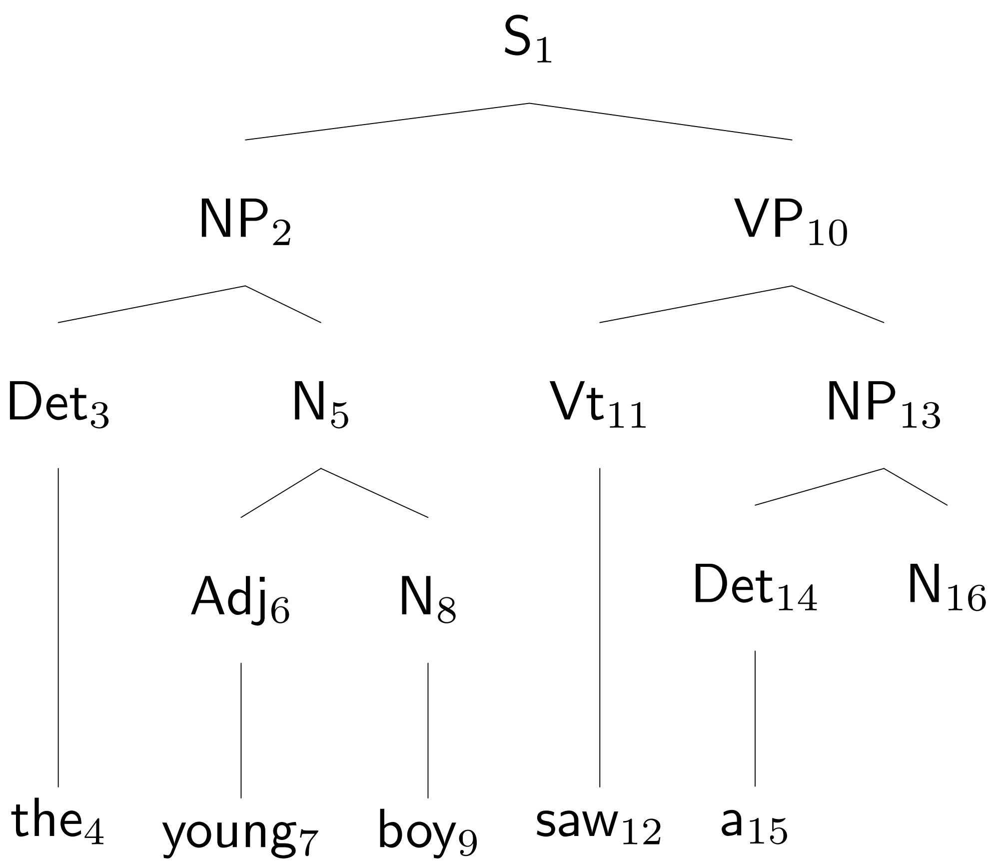


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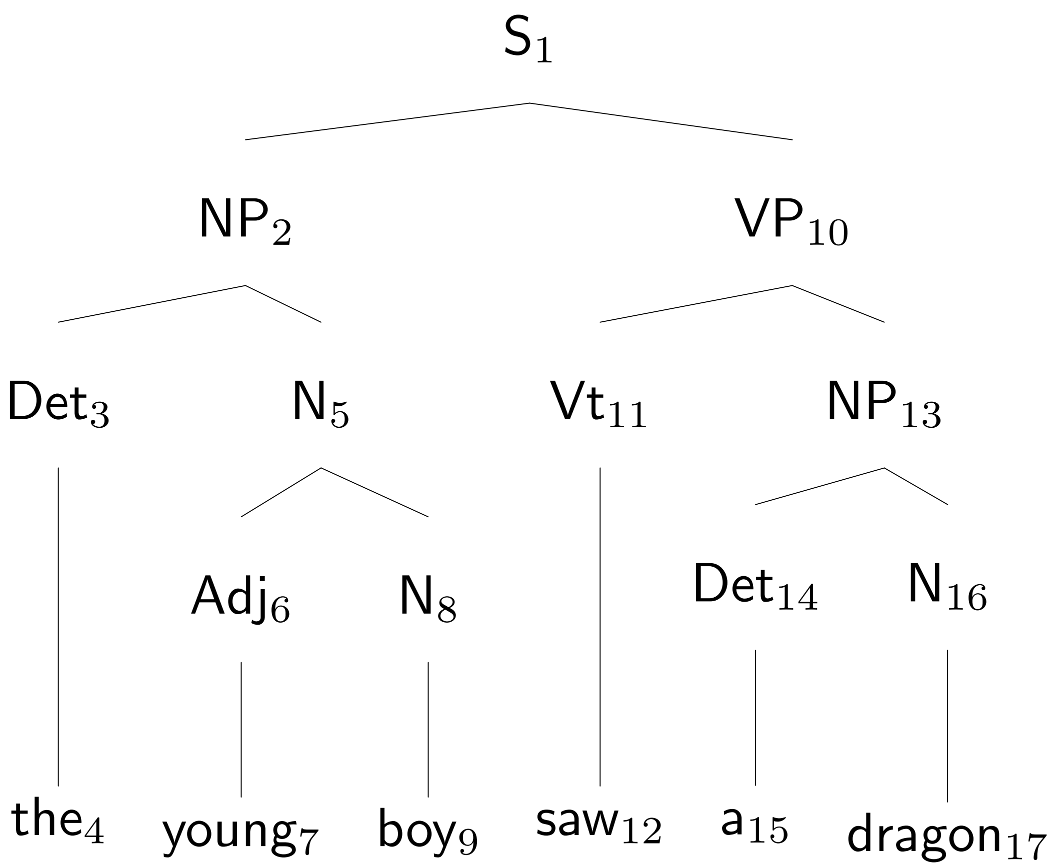


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# A trace (parser/simple/grammar.pl, parser/simple/td_parser_trace.pl)

```
?- td_parse([the,young,boy,saw,the,dragon]).
< [s], [the, young, boy, saw, the, dragon] >
< [np, vp], [the, young, boy, saw, the, dragon] >
< [det, n, vp], [the, young, boy, saw, the, dragon] >
< [the, n, vp], [the, young, boy, saw, the, dragon] >
< [n, vp], [young, boy, saw, the, dragon] >
< [dragon, vp], [young, boy, saw, the, dragon] >
< [boy, vp], [young, boy, saw, the, dragon] >
< [adj, n, vp], [young, boy, saw, the, dragon] >
< [young, n, vp], [young, boy, saw, the, dragon] >
< [n, vp], [boy, saw, the, dragon] >
< [dragon, vp], [boy, saw, the, dragon] >
< [boy, vp], [boy, saw, the, dragon] >
< [vp], [saw, the, dragon] >
```

```
< [vt, np], [saw, the, dragon] >
< [saw, np], [saw, the, dragon] >
< [np], [the, dragon] >
< [det, n], [the, dragon] >
< [the, n], [the, dragon] >
< [n], [dragon] >
< [dragon], [dragon] >
< [], [] >
```

# Bottom-up parsing

- **Start configuration** for recognizing a string $\omega$: $< \epsilon, \omega >$

- **Available actions**:
  - **shift**: turn to the next terminal $a$ of the string
    $< \alpha, a\tau > \mapsto < \alpha a, \tau >$
  - **reduce**: apply a phrase structure rule
    $< \beta\alpha, \tau > \mapsto < \beta A, \tau >$ if $A \rightarrow \alpha \in P$

- **Success configuration**: $< S, \epsilon >$

# A shift-reduce parser in Prolog (parser/simple/sr_parser.pl)

```
:- op(1100,xfx,'--->').

sr_parse(String) :- sr_parse([],String).     % Start

sr_parse([s],[]).                             % Success

sr_parse(Stack,String) :-                     % Reduce
        append(Beta,Alpha,Stack),
        (A ---> Alpha),
        append(Beta,[A],NewStack),
        sr_parse(NewStack,String).

sr_parse(Stack,[Word|String]) :-              % Shift
        append(Stack,[Word],NewStack),
        sr_parse(NewStack,String).
```

# Bottom-up, left-right, depth-first tree traversal

S $\longrightarrow$ NP VP
VP $\longrightarrow$ Vt NP
NP $\longrightarrow$ Det N
N $\longrightarrow$ Adj N

Vt $\longrightarrow$ saw
Det $\longrightarrow$ the
Det $\longrightarrow$ a
N $\longrightarrow$ dragon
N $\longrightarrow$ boy
Adj $\longrightarrow$ young

the$_1$

# Bottom-up, left-right, depth-first tree traversal

$\text{Det}_2$

$\text{the}_1$

S $\longrightarrow$ NP VP
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N $\longrightarrow$ Adj N

Vt $\longrightarrow$ saw
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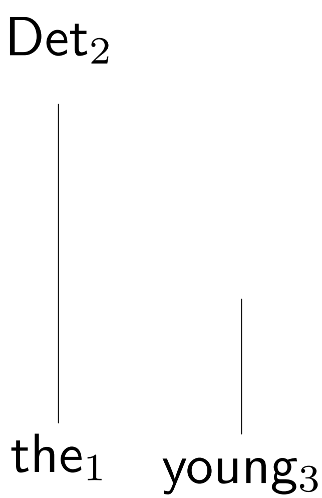
# Bottom-up, left-right, depth-first tree traversal

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# Bottom-up, left-right, depth-first tree traversal

$\text{Det}_2$ $\quad$ $\text{Adj}_4$

$\text{the}_1$ $\quad$ $\text{young}_3$

S $\longrightarrow$ NP VP
VP $\longrightarrow$ Vt NP
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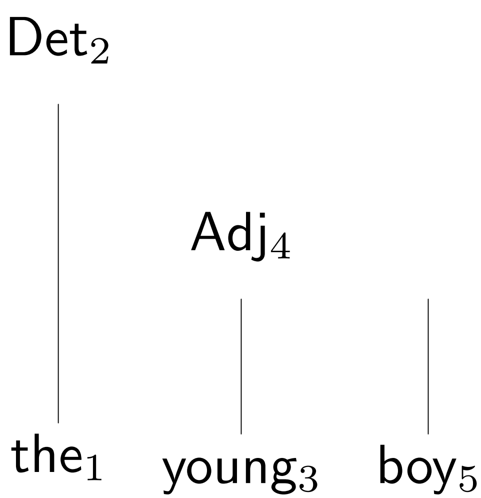
## Bottom-up, left-right, depth-first tree traversal

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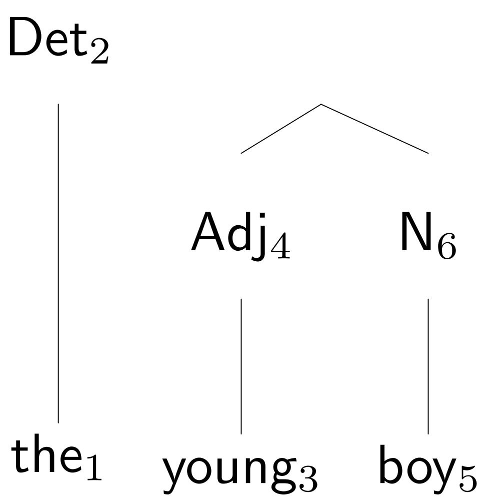


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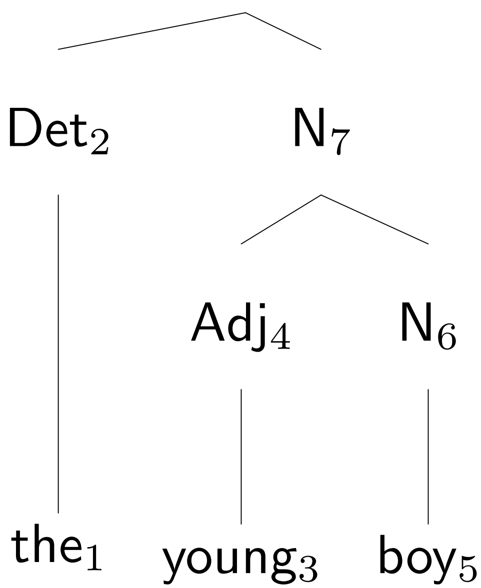
# Bottom-up, left-right, depth-first tree traversal

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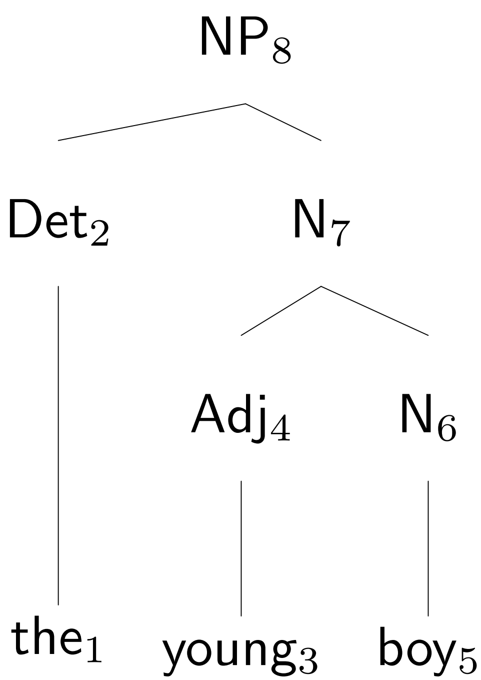


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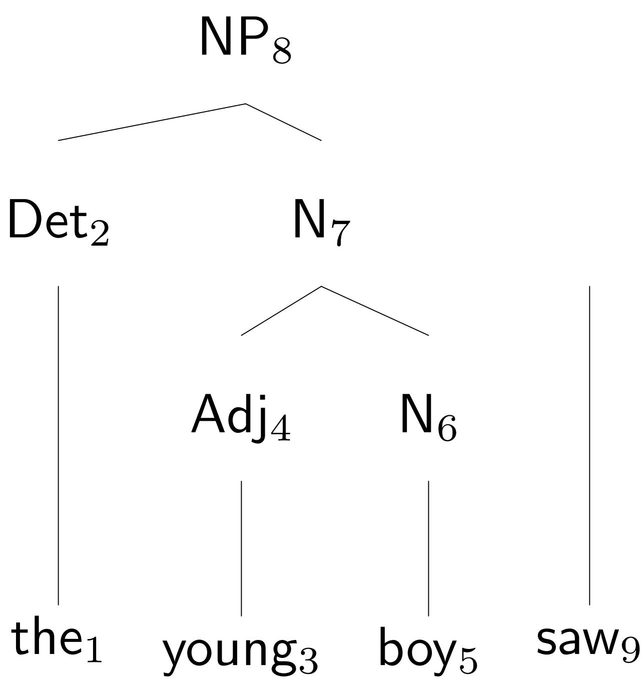


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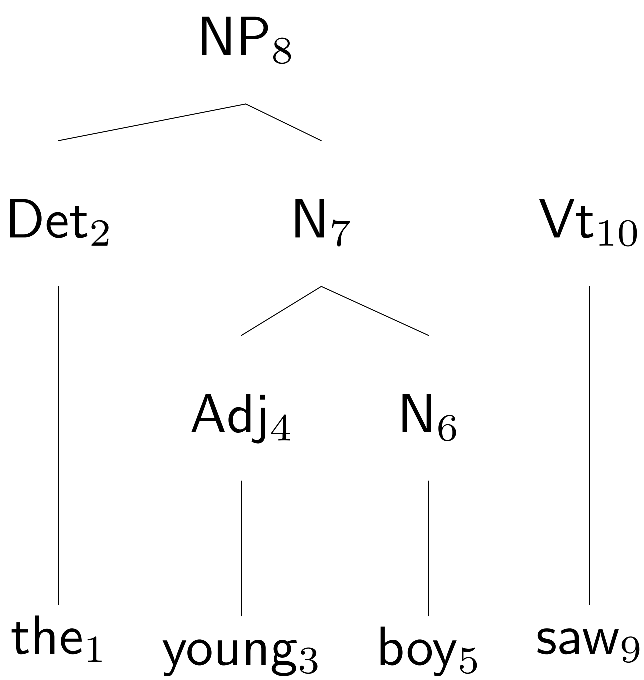


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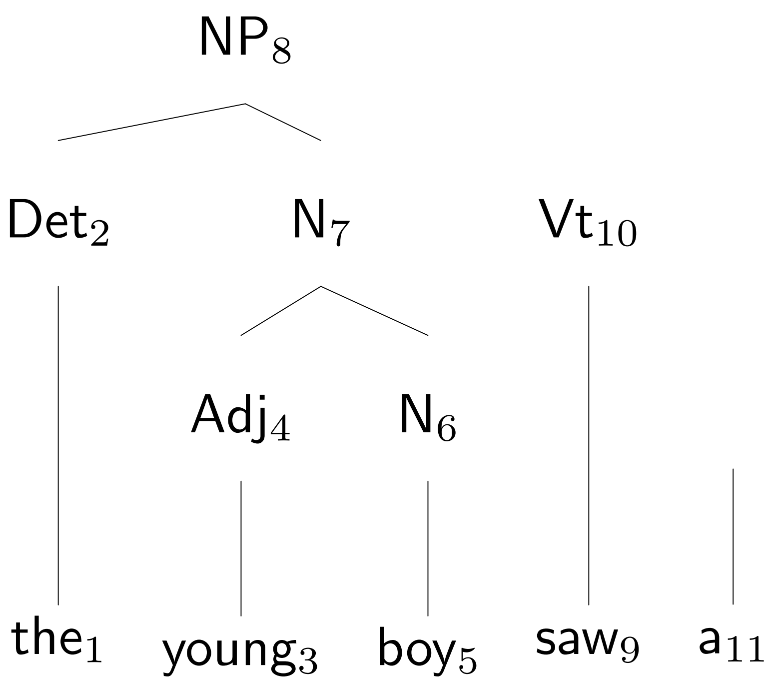


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$Adj \rightarrow young$

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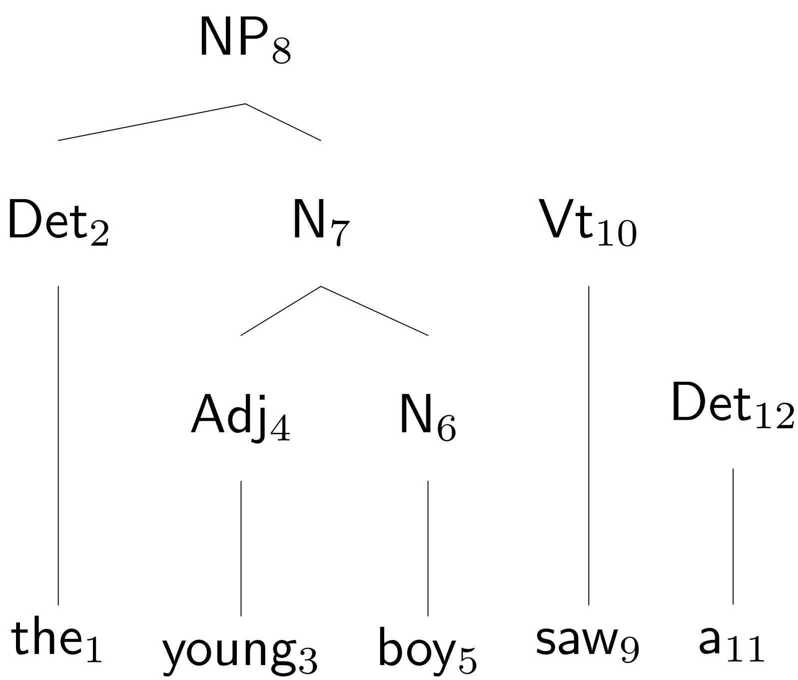


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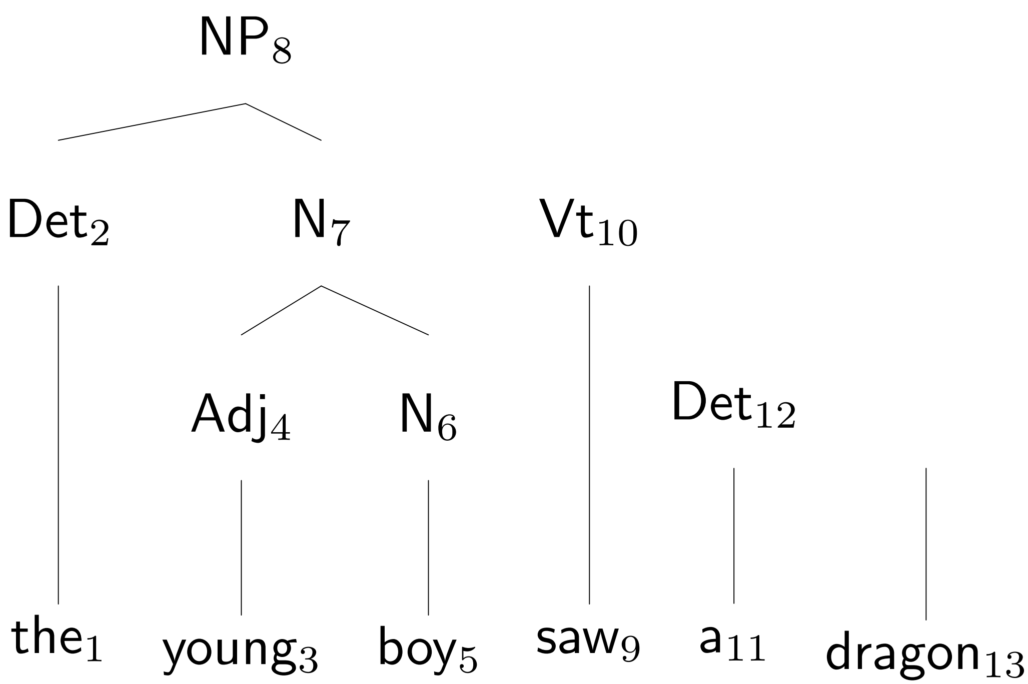


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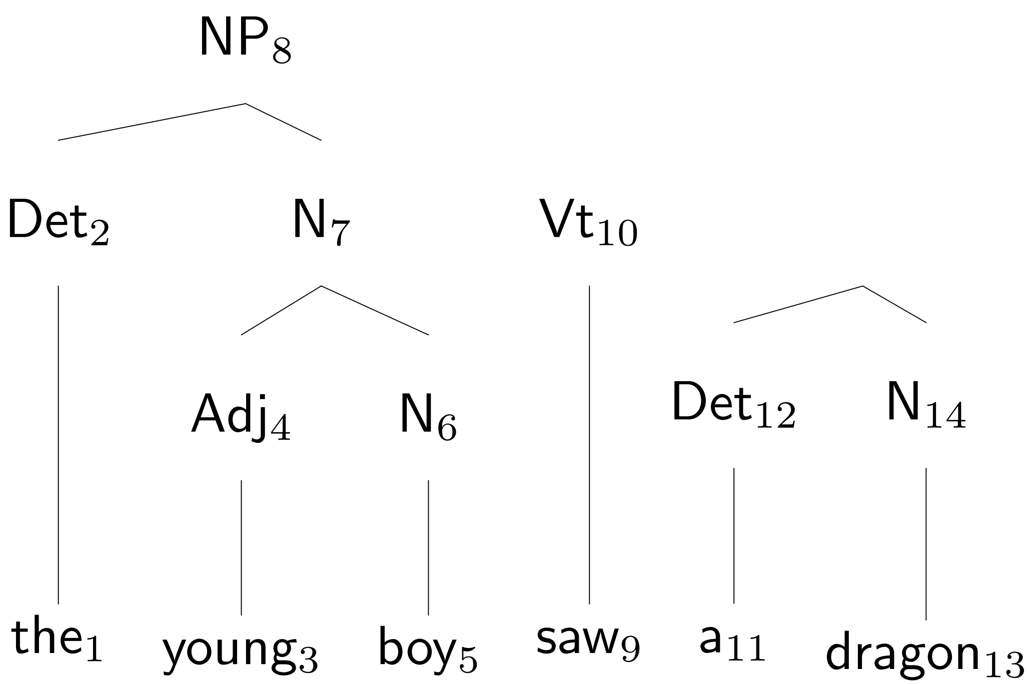


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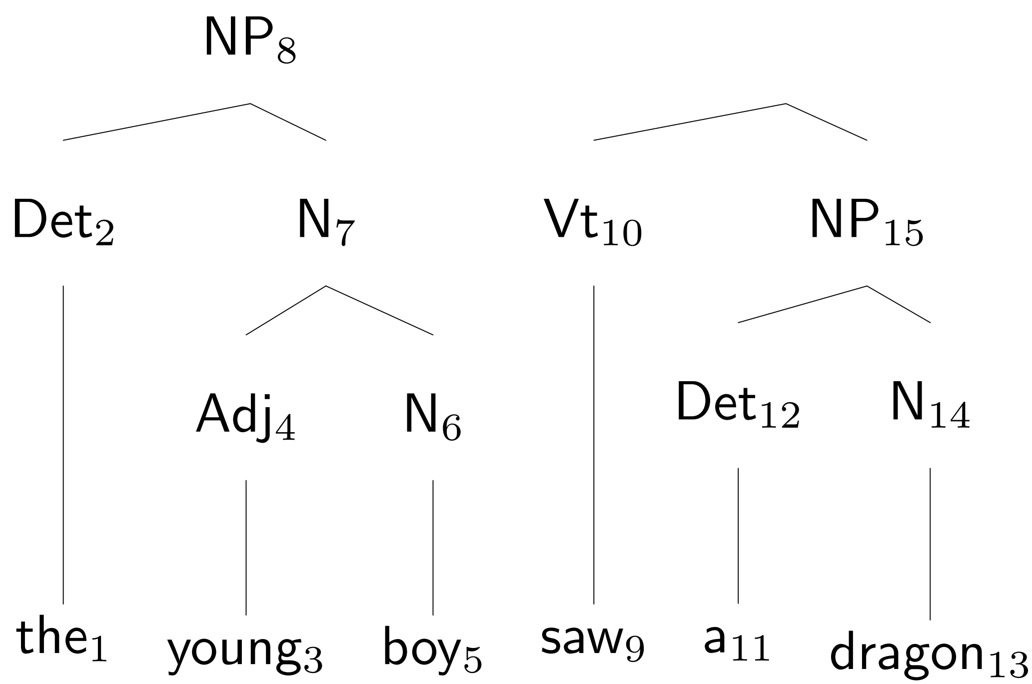
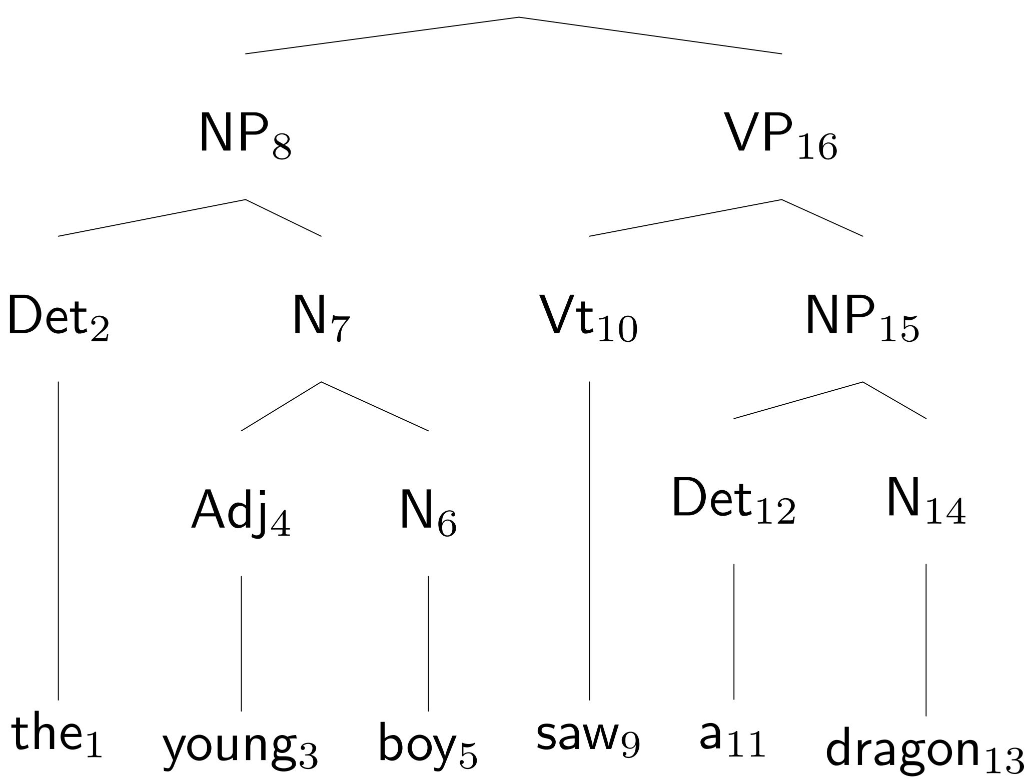
# Bottom-up, left-right, depth-first tree traversal




S $\rightarrow$ NP VP
VP $\rightarrow$ Vt NP
NP $\rightarrow$ Det N
N $\rightarrow$ Adj N

Vt $\rightarrow$ saw
Det $\rightarrow$ the
Det $\rightarrow$ a
N $\rightarrow$ dragon
N $\rightarrow$ boy
Adj $\rightarrow$ young

# Bottom-up, left-right, depth-first tree traversal

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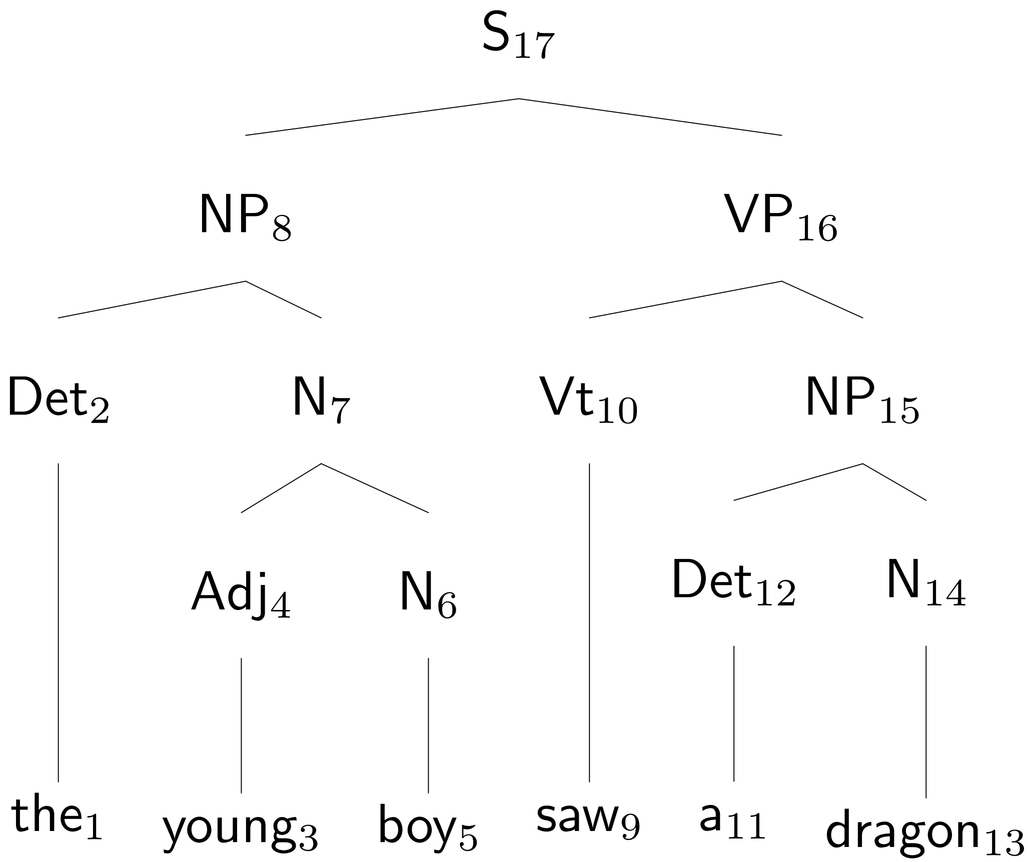


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# A trace (parser/simple/grammar.pl, parser/simple/sr_parser_trace.pl)

```
| ?- sr_parse([the,young,boy,saw,the,dragon]).
START: <[],[the,young,boy,saw,the,dragon]>
     Reduce []? no
     Shift "the"
<[the],[young,boy,saw,the,dragon]>
     Reduce [the] => det
<[det],[young,boy,saw,the,dragon]>
     Reduce [det]? no
     Reduce []? no
     Shift "young"
<[det,young],[boy,saw,the,dragon]>
     Reduce [det,young]? no
     Reduce [young] => adj
```

```
<[det,adj],[boy,saw,the,dragon]>
     Reduce [det,adj]? no
     Reduce [adj]? no
     Reduce []? no
     Shift "boy"
<[det,adj,boy],[saw,the,dragon]>
     Reduce [det,adj,boy]? no
     Reduce [adj,boy]? no
     Reduce [boy] => n
<[det,adj,n],[saw,the,dragon]>
     Reduce [det,adj,n]? no
     Reduce [adj,n] => n
<[det,n],[saw,the,dragon]>
     Reduce [det,n] => np
<[np],[saw,the,dragon]>
     Reduce [np]? no
     Reduce []? no
     Shift "saw"
```

```
<[np,saw],[the,dragon]>
     Reduce [np,saw]? no
     Reduce [saw] => vt
<[np,vt],[the,dragon]>
     Reduce [np,vt]? no
     Reduce [vt]? no
     Reduce []? no
     Shift "the"
<[np,vt,the],[dragon]>
     Reduce [np,vt,the]? no
     Reduce [vt,the]? no
     Reduce [the] => det
<[np,vt,det],[dragon]>
     Reduce [np,vt,det]? no
     Reduce [vt,det]? no
     Reduce [det]? no
     Reduce []? no
     Shift "dragon"
```

```
<[np,vt,det,dragon],[]>
     Reduce [np,vt,det,dragon]? no
     Reduce [vt,det,dragon]? no
     Reduce [det,dragon]? no
     Reduce [dragon] => n
<[np,vt,det,n],[]>
     Reduce [np,vt,det,n]? no
     Reduce [vt,det,n]? no
     Reduce [det,n] => np
<[np,vt,np],[]>
     Reduce [np,vt,np]? no
     Reduce [vt,np] => vp
<[np,vp],[]>
     Reduce [np,vp] => s
<[s],[]>
SUCCESS!
```

# A shift-reduce parser for grammars in CNF using difference lists to encode the string (parser/simple/cnf_sr.pl)

```
:- op(1100,xfx,'--->').

recognise(String) :- recognise([],String) % Start

recognise([s],[]).             % Success

recognise([Y,X|Rest],S) :-     % Reduce
    (LHS ---> [X,Y]),
    recognise([LHS|Rest],S).

recognise(Stack,[Word|S]) :- % Shift
    Cat ---> [Word],
    recognise([Cat|Stack],S).
```

## A trace (parser/simple/grammar.pl, parser/simple/cnf_sr_trace.pl)

```
| ?- recognise([the,young,boy,saw,the,dragon]).
START: <[],[the,young,boy,saw,the,dragon]>
     Shift "the" as "det"
<[det],[young,boy,saw,the,dragon]>
     Shift "young" as "adj"
<[adj,det],[boy,saw,the,dragon]>
     Reduce [det,adj]? no
     Shift "boy" as "n"
<[n,adj,det],[saw,the,dragon]>
     Reduce [adj,n] => n
<[n,det],[saw,the,dragon]>
     Reduce [det,n] => np
<[np],[saw,the,dragon]>
     Shift "saw" as "vt"
```

```
<[vt,np],[the,dragon]>
     Reduce [np,vt]? no
     Shift "the" as "det"
<[det,vt,np],[dragon]>
     Reduce [vt,det]? no
     Shift "dragon" as "n"
<[n,det,vt,np],[]>
     Reduce [det,n] => np
<[np,vt,np],[]>
     Reduce [vt,np] => vp
<[vp,np],[]>
     Reduce [np,vp] => s
<[s],[]>
SUCCESS!
```