Remembering subresults (Part I):
Well-formed substring tables

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Problem: Inefficiency of recomputing subresults

Two example sentences and their potential analysis:

(1) He [gave [the young cat] [to Bill]].
(2) He [gave [the young cat] [some milk]].

The corresponding grammar rules:

```
vp ---> [v_ditrans, np, pp_to].
vp ---> [v_ditrans, np, np].
```


Solution: Memoization

- Store intermediate results:
  a) completely analyzed constituents:
     well-formed substring table or **(passive) chart**
  b) partial and complete analyses:
     (active) chart
- All intermediate results need to be stored for completeness.
- All possible solutions are explored in parallel.

CFG Parsing: The Cocke Younger Kasami Algorithm

- Grammar has to be in Chomsky Normal Form (CNF), only
  - RHS with a single terminal: $A \rightarrow a$
  - RHS with two non-terminals: $A \rightarrow BC$
  - no ϵ rules ($A \rightarrow \epsilon$)
- A representation of the string showing positions and word indices:

  $\cdot_0\ w_1\ \cdot_1\ w_2\ \cdot_2\ w_3\ \cdot_3\ w_4\ \cdot_4\ w_5\ \cdot_5\ w_6\ \cdot_6$

  For example: $\cdot_0$ the $\cdot_1$ young $\cdot_2$ boy $\cdot_3$ saw $\cdot_4$ the $\cdot_5$ dragon $\cdot_6$

The well-formed substring table (= passive chart)

- The well-formed substring table, henceforth (passive) chart, for a string of length n is an $n \times n$ matrix.
- The field (i, j) of the chart encodes the set of all categories of constituents that start at position i and end at position j, i.e.
  chart(i,j) = $\{A \mid A \Rightarrow^* w_{i+1} \dots w_j\}$
- The matrix is triangular since no constituent ends before it starts.

Coverage Represented in the Chart

An input sentence with 6 words:

$\cdot_0\ w_1\ \cdot_1\ w_2\ \cdot_2\ w_3\ \cdot_3\ w_4\ \cdot_4\ w_5\ \cdot_5\ w_6\ \cdot_6$

Coverage represented in the chart:

TO:

FROM:	1	2	3	4	5	6
0	0–1	0–2	0–3	0–4	0–5	0–6
1		1–2	1–3	1–4	1–5	1–6
2			2–3	2–4	2–5	2–6
3				3–4	3–5	3–6
4					4–5	4–6
5						5–6

Example for Coverage Represented in Chart

Example sentence:

$\cdot_0$ the $\cdot_1$ young $\cdot_2$ boy $\cdot_3$ saw $\cdot_4$ the $\cdot_5$ dragon $\cdot_6$

Coverage represented in chart:

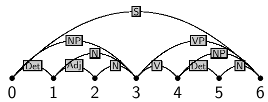
	1	2	3	4	5	6
0	the	the young	the young boy	the young boy saw	the young boy saw the	the young boy saw the dragon
1		young	young boy	young boy saw	young boy saw the	young boy saw the dragon
2			boy	boy saw	boy saw the	boy saw the dragon
3				saw	saw the	saw the dragon
4					the	the dragon
5						dragon

An Example for a Filled-in Chart

Input sentence:
$\cdot_0$ the $\cdot_1$ young $\cdot_2$ boy $\cdot_3$ saw $\cdot_4$ the $\cdot_5$ dragon $\cdot_6$

Chart:

	1	2	3	4	5	6
0	{Det}	{}	{NP}	{}	{}	{S}
1		{Adj}	{N}	{}	{}	{}
2			{N}	{}	{}	{}
3				{V, N}	{}	{VP}
4					{Det}	{NP}
5						{N}



Grammar:
S → NP VP
VP → Vt NP
NP → Det N
N → Adj N
Vt → saw
Det → the
Det → a
N → dragon
N → boy
N → saw
Adj → young

Filling in the Chart

- It is important to fill in the chart systematically.
- We build all constituents that end at a certain point before we build constituents that end at a later point.

	1	2	3	4	5	6
0	1	3	6	10	15	21
1		2	5	9	14	20
2			4	8	13	19
3				7	12	18
4					11	17
5						16

```
for j := 1 to length(string)
    lexical_chart_fill(j − 1, j)
    for i := j − 2 down to 0
        syntactic_chart_fill(i, j)
```

lexical_chart_fill(j-1,j)

- Idea: Lexical lookup. Fill the field $(j-1, j)$ in the chart with the preterminal category dominating word j.
- Realized as:

$$chart(j-1, j) := \{X \mid X \rightarrow \text{word}_j \in P\}$$

syntactic_chart_fill(i,j)

- Idea: Perform all reduction step using syntactic rules such that the reduced symbol covers the string from i to j.
- Realized as: $chart(i,j) = \left\{ A \;\middle|\; \begin{array}{l} A \rightarrow BC \in P, \\ i < k < j, \\ B \in chart(i,k), \\ C \in chart(k,j) \end{array} \right\}$
- Explicit loops over every possible value of k and every context free rule:

$chart(i,j) := \{\}.$
for $k := i+1$ to $j-1$
  for every $A \rightarrow BC \in P$
    if $B \in chart(i,k)$ and $C \in chart(k,j)$ then
      $chart(i,j) := chart(i,j) \cup \{A\}.$

The Complete CYK Algorithm

Input: start category S and input $string$

$n :=$ length($string$)

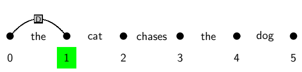
for $j := 1$ to n
  $chart(j-1,j) := \{X \mid X \rightarrow \text{word}_j \in P\}$
  for $i := j-2$ down to 0
    $chart(i,j) := \{\}$
    for $k := i+1$ to $j-1$
      for every $A \rightarrow BC \in P$
        if $B \in chart(i,k)$ and $C \in chart(k,j)$ then
          $chart(i,j) := chart(i,j) \cup \{A\}$

Output: if $S \in chart(0,n)$ then accept else reject

Example Application of the CYK Algorithm

s → np vp d → the Lexical Entry: *the* ($j = 1$, field chart(0,1))
np → d n n → dog
vp → v np n → cat
v → chases

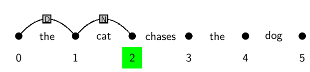
	1	2	3	4	5
0	d				
1					
2					
3					
4					



Example Application of the CYK Algorithm

s → np vp d → the Lexical Entry: *cat* ($j = 2$, field chart(1,2))
np → d n n → dog
vp → v np n → cat
v → chases

	1	2	3	4	5
0	d				
1		n			
2					
3					
4					

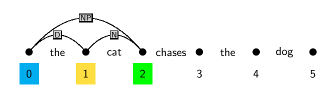


Example Application of the CYK Algorithm

s → np vp d → the
np → d n n → dog
vp → v np n → cat
v → chases

$j = 2$
$i = 0$
$k = 1$

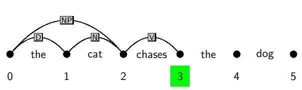
	1	2	3	4	5
0	d	np			
1		n			
2					
3					
4					



Example Application of the CYK Algorithm

s → np vp d → the Lexical Entry: *chases* ($j = 3$, field chart(2,3))
np → d n n → dog
vp → v np n → cat
v → chases

	1	2	3	4	5
0	d	np			
1		n			
2			v		
3					
4					

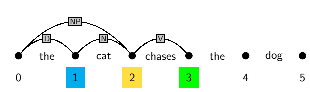


Example Application of the CYK Algorithm

s → np vp d → the
np → d n n → dog
vp → v np n → cat
v → chases

$j = 3$
$i = 1$
$k = 2$

	1	2	3	4	5
0	d	np			
1		n			
2			v		
3					
4					

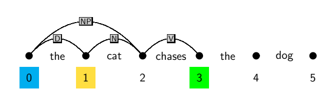


Example Application of the CYK Algorithm

s → np vp d → the
np → d n n → dog
vp → v np n → cat
v → chases

$j = 3$
$i = 0$
$k = 1$

	1	2	3	4	5
0	d	np			
1		n			
2			v		
3					
4					



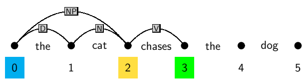
Example Application of the CYK Algorithm

s → np vp
np → d n
vp → v np

d → the
n → dog
n → cat
v → chases

j = 3
i = 0
k = 2

	1	2	3	4	5
0	d	np			
1		n			
2			v		
3					
4					



Dynamic knowledge bases in PROLOG

- Declaration of a dynamic predicate: `dynamic/1` declaration, e.g:

  `:- dynamic chart/3.`

  to store facts of the form `chart(From,To,Category)`:

- Add a fact to the database: `assert/1`, e.g.:

  `assert(chart(1,3,np)).`

  Special versions `asserta/1`/`assertz/1` ensure adding facts first/last.

- Removing a fact from the database: `retract/1`, e.g.:

  `retract(chart(1,_,np)).`

  To remove all matching facts from the database use `retractall/1`

The CYK algorithm in PROLOG (parser/cky/cky.pl)

```
:- dynamic chart/3.            % chart(From,To,Category)
:- op(1100,xfx,'--->').        % Operator for grammar rules


% recognize(+WordList,?Startsymbol): top-level of CYK recognizer

recognize(String,Cat) :-
   retractall(chart(_,_,_)),    % initialize chart
   length(String,N),            % determine length of string
   fill_chart(String,0,N),      % call parser to fill the chart
   chart(0,N,Cat).              % check whether parse successful
```



```
% fill_chart(+WordList,+Current minus one,+Last)
% J-LOOP from 1 to n

fill_chart([],N,N).
fill_chart([W|Ws],JminOne,N) :-
   J is JminOne + 1,
   lexical_chart_fill(W,JminOne,J),
   %
   I is J - 2,
   syntactic_chart_fill(I,J),
   %
   fill_chart(Ws,J,N).
```



```
% lexical_chart_fill(+Word,+JminOne,+J)
% fill diagonal with preterminals

lexical_chart_fill(W,JminOne,J) :-
   (Cat ---> [W]),
   add_to_chart(JminOne,J,Cat),
   fail
 ; true.
```



```
% syntactic_chart_fill(+I,+J)
% I-LOOP from J-2 downto 0

syntactic_chart_fill(-1,_) :- !.
syntactic_chart_fill(I,J) :-
   K is I+1,
   build_phrases_from_to(I,K,J),
   %
   IminOne is I-1,
   syntactic_chart_fill(IminOne,J).
```



```
% build_phrases_from_to(+I,+Current-K,+J)
% K-LOOP from I+1 to J-1

build_phrases_from_to(_,J,J) :- !.
build_phrases_from_to(I,K,J) :-
   chart(I,K,B),
   chart(K,J,C),
   (A ---> [B,C]),
   add_to_chart(I,J,A),
   fail
 ; KplusOne is K+1,
   build_phrases_from_to(I,KplusOne,J).
```



```
% add_to_chart(+Cat,+From,+To): add if not yet there
add_to_chart(From,To,Cat) :-
   chart(From,To,Cat),
   !.
add_to_chart(From,To,Cat) :-
   assertz(chart(From,To,Cat)).
```